Financial Literacy and Precautionary Insurance

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Abstract

This paper studies insurance demand when individuals exhibit limited financial literacy. Financially illiterate individuals are uncertain about the payout of complex insurance contracts. We show that a trade off between second-order and third-order risk preferences drives insurance demand. Sufficiently prudent individuals increase insurance demand with more complex contracts, while the effect is reversed for less prudent individuals. Under reasonable conditions, a positive level of contract complexity exists in competitive market equilibrium. We quantify the welfare loss from financial illiteracy, which amounts to 1-3\% of wealth under reasonable assumptions. We provide a novel rationale for individual decision-making under risk with financially illiterate consumers and discuss implications for welfare and consumer protection.

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1 Introduction

Many financial products confront consumers with complex information. This is particularly the case for insurance contracts, which often include legalese language (Cogan (2010)) that is rarely fully understood by consumers (Policygenius (2016), The Guardian Life Insurance Company of America (2017), Fairer Finance (2018)). At the same time, we observe low levels of financial literacy across large parts of the population worldwide (Lusardi and Mitchell (2011a), Lusardi and Mitchell (2014)), indicating a low ”ability to process economic information and make informed decisions” (Behrman et al. (2012)). For example, only half of the U.S. population reads at the basic level1 and financial planning competence varies substantially by age and gender2 However, although financially illiterate consumers are confronted with highly complex insurance contracts in practice, research on the impact of financial literacy on insurance demand is very scarce3

To address this gap in the literature, this paper presents a novel approach to understand insurance decisions of financially illiterate individuals. It then discusses implications for market equilibria in an expected utility framework. Motivated by the empirical observation that consumers rarely fully understand insurance contracts, the main idea of the model is that contract complexity results in an information friction for financially illiterate individuals. Contract complexity is modeled as an individual’s uncertainty about the insurance indemnity payment. Contract complexity is then similar to (exogenous) background risk, yet distinct since it becomes endogenous to the insurance contract: the higher the insurance coverage, the larger is the variability of the insurance payoff experienced by individuals.4

Results show that financial illiteracy heavily alters insurance decisions. A precautionary insurance motive arises for sufficiently prudent individuals, who prepare for a higher perceived risk (stemming from contract complexity) by increasing wealth in the worst possible state. A positive level of contract complexity exists in a competitive equilibrium if companies face high

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1See the 2002 literacy survey of the U.S. Department of Education: Sum et al. (2002).
2Lusardi and Mitchell (2008) find that women are typically less financially literate than men, based on questions about interest compounding, inflation, and risk diversification.
3To the best of our knowledge, Gaurav et al. (2011) is the one exception. The authors empirically examine the impact of financial literacy education on the demand for rainfall insurance in a field experiment in rural India.
4Note that we do not model financial illiteracy as a wealth effect because not fully understanding a contract is not necessarily the same thing as having a negative bias about the payout.
transparency cost, making it costly to reduce contract complexity. Based on the equilibrium analysis, we study the welfare cost of financial illiteracy, referred to as the financial illiteracy premium. Under reasonable conditions, the financial illiteracy premium amounts to 1% to 3% of individuals’ endowment, highlighting the relevance of financial illiteracy for welfare. This result shows that financial illiteracy reduces welfare in competitive markets. However, in reality, insurance markets often exhibit oligopolistic structures, and thus firms might exploit market power to offer products at inefficiently high prices and/or high contract complexity. Therefore, it seems reasonable to expect that welfare cost of financial illiteracy are even larger in practice.

This study contributes to the increasing literature on financial illiteracy (also referred to as investor unsophistication) and information frictions in financial markets. Delavande et al. (2008), Jappelli and Padula (2013), Kim et al. (2016), Lusardi et al. (2017), and Neumuller and Rothschild (2017) study portfolio choice for individuals with information frictions. For example, Anagol and Kim (2012) show that mutual funds attract unsophisticated investors by lacking clarity in pricing. Our modeling of imperfect information is most closely related to the one of Neumuller and Rothschild (2017), in which individuals receive imperfect signals about the characteristics of investment opportunities.

Previous studies are extended mainly along three lines. First, we contribute to the literature by focusing on insurance contracts, in particular, since these are among the most utilized financial and risk management products. Moreover, they seem particularly complex and not well-understood by consumers (see Section 2). Second, we provide a comprehensive analysis on the dependence between financial products and risk attitudes, providing a more granular understanding of the behavior of financially illiterate individuals. Third, we provide a general equilibrium analysis that yields insights into how financial illiteracy impacts the supply of financial contracts.

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5 Transparency cost can result, e.g., from operational costs to create additional documentation.

6 To derive this baseline result, we assume that individuals endowed with initial wealth of $100 maximize exponential utility with constant absolute risk aversion 0.02 for a loss of $50 that occurs with probability 30%. We provide a sensitivity analysis for this result in Section 4.

7 For example, according to the National Association of Insurance Commissioners (NAIC) (2017), the largest 5 insurers had a joint market share of more than 30% of the U.S. and Canadian property & casualty insurance market in 2017. In the total private passenger auto insurance market, 4 insurers had a joint market share of more than 50% in 2017.

8 Car, life, and private health insurance are among the top six financial products and services acquired by European citizens (the other three are a current bank or savings account and a credit card; TNS opinion & social (2016)).
The results and the model framework of this study are not limited to the insurance market or the assumption of rational individuals. In contrary, the present paper provides a general tool for modeling financial illiteracy that can be applied in numerous other financial decisions, such as decisions about optimal portfolio investments or optimal saving. Indeed, since insurance results in a wealth transfer from good to bad states, one can also interpret insurance as a savings product that hedges (e.g., low future income), with the states being two different points in time. Furthermore, it is straightforward to include other behavioral phenomena such as ambiguity aversion. In this case, ambiguity aversion can be interpreted as disutility from contract complexity, since we model contract complexity via an increase in second-order uncertainty.

The remainder of this article is organized as follows. In the following section, we relate this study to the previous literature and provide a background on financial literacy. Section 3 introduces our model and derives baseline results. Section 4 adds an equilibrium model and introduces and discusses the concept of a financial illiteracy premium. The final section concludes.

2 Background and Related Literature

Several studies provide robust empirical evidence of low financial literacy levels globally, as shown by Lusardi and Mitchell (2011a) and Sum et al. (2002). Financial literacy levels are of public concern as economic outcomes are highly dependent on financial literacy: Lusardi and Mitchell (2007) and Lusardi and Mitchell (2011b) find a profound impact of financial (il-)literacy on an individual’s ability to plan. Individuals with low financial literacy are found to be more likely to have problems with debt (Lusardi and Tufano (2015)), make inefficient portfolio choices (Van Rooij et al. (2011), Hastings and Tejeda-Ashton (2008), Guiso and Jappelli (2009)), accumulate and manage wealth less effectively (Stango and Zinman (2007), Hilgert and Beverly (2003)), and use revolving consumer credit with high interest charges even in cases when they could immediately pay down all debt using their liquid assets (Gathergood and Weber (2014)).

There is ample evidence, in particular, that consumers do not fully understanding their insurance contracts across almost all lines of insurance, e.g. reported by Quantum Market.

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9Even though, culture seems to impact levels of financial literacy, see e.g. Brown et al. (2018).
Research for the Insurance Council of Australia (2013) for Australian home insurance policies, Policygenius (2016) for U.S. health plans, Davidoff et al. (2017) for reverse mortgages, The Guardian Life Insurance Company of America (2017) for U.S. employee benefits packages. Fairer Finance (2018) describe numerous situations in which individuals are unaware of the specific risks covered under their insurance policy. One potential reason for illiteracy about insurance contracts is that insurance naturally pays out only in case of a loss, which is usually a low probability event. Hence, the return of insurance seems less easy to evaluate than that of many other financial products, as e.g. equity investments. Indeed, several studies provide empirical and experimental evidence that individuals exhibit substantial behavioral biases and high estimation errors when evaluating risks. Additionally, a large number of studies concedes that individuals do not read their insurance contracts at all (White and Mansfield (2002), Ben-Shahar (2009), Becher and Unger-Avivram (2010), Cogan (2010), Eigen (2012)). These empirical observations motivate our model of financial illiteracy.

Financial service providers could theoretically invest in decreasing the complexity of offered products, which may be specifically beneficial for less financially literate consumers. Yet, empirical and theoretical studies suggest that they face little incentives to do so. Several studies find that financial firms exploit unsophisticated consumers via unclear pricing methods (DellaVigna and Malmendier (2004), Gabaix and Laibson (2006), Anagol and Kim (2012), and Campbell (2016)), and that financially less literate consumers end up paying relatively higher prices because they do not completely understand the products’ price structure (Carlin (2009)). Carlin (2009) models consumers to either be financial experts who choose the objectively optimal product, or uninformed customers who choose randomly. Our approach extends his model by explicitly varying the level of consumers’ unsophistication. Therefore, the present paper evaluates more granular differences in sophistication and their impact on consumer behavior. In the model of Acharya and Bisin (2014), complexity about derivatives contracts arises as entities are uninformed about the risk of their counterparties. Similarly to our model of insurance decisions, the dealers in Acharya and Bisin (2014)’s model engage in excessive risk-taking, that reduces overall welfare. Nudging individuals to obtain financial advice (Kramer (2016)) or investing into financial literacy education (Meier and Sprenger, 2016). Kühn et al. (1978) find that individuals refrain from buying flood insurance even when it is greatly subsidized and priced below its actuarially fair value. Johnson et al. (1993) provide experimental evidence that consumers exhibit distortions in their perception of risk, as well as framing effects in evaluating premiums and benefits. More generally, Kahneman and Tversky (1979) show that individuals often overweight small probabilities.
have been mentioned to address issues of adverse economic outcomes for financially less literate individuals.

The most closely related literature to this study examines the impact of financial illiteracy on financial decision-making by the means of a theoretical model. Previous studies by Delavande et al. (2008), Jappelli and Padula (2013), Kim et al. (2016), Lusardi et al. (2017), and Neu-muller and Rothschild (2017) predominantly focus on portfolio choice in partial equilibrium with fixed supply. We extend these studies by providing an in-depth analysis of insurance contracts, endogenizing product complexity in a competitive equilibrium setting. This study contributes to the insurance economics literature. Most insurance models interpret an insurance contract as a pair of only two parameters, namely the insurance premium paid by the insured and the indemnity payment paid by the firm in case of a loss (e.g. see Doherty (1975)). This study introduces contract complexity as a third characteristic of insurance contracts. Lee (2012) studies the impact of uncertain indemnity payments on insurance demand, and thereby resembles our modeling approach for financial illiteracy. His main result is that partial coverage is optimal in the presence of uncertain indemnity payments if prudence is not too large. We extend his result by providing comparative statics for the variability of indemnity payments, endogenizing variable indemnity payments in a general equilibrium framework, and relating it to the cost of financial illiteracy.

Since we interpret contract complexity as a risk attached to an insurance contract’s payout, our model also relates to studies on insurance nonperformance, i.e., default risk. Insurance demand in the presence of default risk is studied, e.g., by Kahneman and Tversky (1979), Tapiero et al. (1986), Doherty and Schlesinger (1990), Briys et al. (1991), Wakker et al. (1997), Zimmer et al. (2018). Default risk substantially differs from contract complexity: It involves both a wealth and a risk effect with lower expected and more variable payout for higher default risk. In contrast, contract complexity in our model only involves a risk effect since it originates from an individual’s uncertainty (but not bias) about payouts. Typically, the comparative static of insurance demand with respect to contract non-performance is ambiguous (e.g., Doherty and Schlesinger (1990), Mahul and Wright (2007)), while our model provides unambiguous comparative statics with respect to contract complexity.

Our article also relates to insurance models with background risk: For fixed insurance coverage, contract complexity can be interpreted as an uninsurable background risk to the individual’s wealth in the loss state. As shown by Fei and Schlesinger (2008), prudent individuals increase insurance coverage upon the introduction of an uninsurable and coverage-independent
background risk in the loss state. Eeckhoudt and Kimball (1992) introduce the term precautionary insurance to describe a prudent individual’s response of increasing insurance coverage when faced with background risk. The general idea in these models is that insurance raises the worst possible wealth as a response to prepare for additional risk. In contrast to background risk, contract complexity is not independent from insurance coverage but rather an inherent feature of the latter. Therefore, our model together with its general results substantially differs from background-risk models. For example, Fei and Schlesinger (2008) show that prudence is sufficient for optimal insurance coverage to increase with coverage-independent background risk in the loss state. We show that prudence alone is not sufficient for increases in contract complexity which is coverage-dependent, but instead prudence must exceed a certain threshold in order to result in precautionary behavior. In addition, we provide an equilibrium analysis with firms endogenously determining the level of contract complexity, which would not be applicable in a situation with background risk.

3 A Model of Contract Complexity

3.1 Insurance Demand

Individuals of mass one are endowed with initial wealth of $w_0$ and face the risk of a loss $L$. The loss occurs with probability $p$. Individuals are risk averse with a twice differentiable and concave standard utility function $u(\cdot): u'(\cdot) > 0$ and $u''(\cdot) < 0$. Firms offer insurance policies and have financial resources such that they are willing and able to sell any number of contracts that they think will make non-negative expected profit.

We deviate from the standard notation of models for insurance demand (such as used by Doherty (1975)) by modeling insurance through an (potentially financially illiterate) individual’s perspective. Such an individual is likely to know the premium she pays for insurance, but may only have a vague idea about the indemnity to expect in the case of a loss. Accordingly, an insurance contract for our individual is defined by two parameters: (1) the expected indemnity payment $\$i$ (in case the loss occurs) per $\$1$ premium paid, and (2) the contract complexity $\varepsilon > 0$ where we denote by $\tilde{\theta}$ the zero-mean contract complexity risk, $\mathbb{P}(\tilde{\theta} = \varepsilon) = \mathbb{P}(\tilde{\theta} = -\varepsilon) = 1/2$. $i$ is the individual’s subjectively expected unit indemnity

\footnote{Our notation differs from classical models in the sense that we consider the indemnity payment in units of $\$1$ paid. This reflects the interpretation of our model as being from an individual’s perspective where prices are exogenously given by the contract but the individual is uncertain about the final indemnity payment.}
payment conditional on the individual’s current information set. If, from the individual’s perspective, the contract is actuarially fair, it is \( p_i = 1 \). In contrast, it is \( p_i < 1 \) (\( p_i > 1 \)) if the individual expects a loading (discount) on the actuarially fair price. We assume that \( i \geq 1 \), implying that the individual expects to receive at least $1 in case of a loss per $1 premium paid.

Contract complexity induces uncertainty about the indemnity via a zero-mean risk: From an individual’s perspective, purchasing \( \alpha > 0 \) units of insurance coverage results in an indemnity of either \( \alpha(i + \varepsilon) \) or \( \alpha(i - \varepsilon) \) with probability \( 1/2 \), \( \varepsilon > 0 \). The fact that individuals may receive more than \( i \) does not necessarily imply that the insurance payment in this state exceeds the loss, i.e., that \( \alpha(i + \varepsilon) > L \). Insurers might very well restrict insurance payments to not exceed \( L \), in which case individuals would maximize expected utility over \( \alpha \in [0, L/(i + \varepsilon)] \). In the following, we will let individuals choose among unbounded insurance take-up to shed light on the demand for complex insurance contracts in general. Since expected utility will be strictly concave in \( \alpha \), utility-maximizing demand \( \alpha^* > L/(i + \varepsilon) \) will simply imply that individuals demand the highest possible coverage up to \( \alpha^* \).

\[
\begin{align*}
\text{no loss:} & \quad w_0 - \alpha = w_2 \\
\text{loss state:} & \quad \begin{cases} 
    w_0 - L + \alpha(i + \varepsilon - 1) = w_{1,+} \\
    w_0 - L + \alpha(i - \varepsilon - 1) = w_{1,-}
\end{cases}
\end{align*}
\]

Figure 1: Distribution of individuals’ wealth.

Figures 1 and 2 illustrate the resulting distribution of an individual’s wealth. Upon purchasing \( \alpha \) units of insurance, individuals pay the premium $\alpha$. Without contract complexity (\( \varepsilon = 0 \)), individuals receive the indemnity payment $\alpha i$ with certainty in the loss state. Otherwise

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\[12\text{Note that our modeling of complexity is equivalent to the model of two-point ambiguity as suggested by Chew et al. (2017). The model is easily extended to allow for skewed background risk } \varepsilon, \text{ e.g., by reducing the probability of the “good” state with payment } \alpha(i + \varepsilon). \text{ Reducing this probability to zero would make the model equivalent to the non-performance model of Doherty and Schlesinger (1990). Importantly, imposing a bias } E[\varepsilon] \neq 0 \text{ will induce a wealth effect on top of the risk effect of complexity. For example, } E[\varepsilon] < 0 \text{ biases insurance demand toward zero, everything else equal. By focusing only on the risk effect, we also contribute to an understanding of Doherty and Schlesinger (1990)’s result that nonperformance may have an ambiguous effect on insurance demand.}\]
Relative Insurance Coverage \((\alpha_i/L_i)\) and Wealth \((w/w_0)\)

Figure 2: States of wealth for fixed coverage.

Distribution of individuals’ wealth with changing level of relative insurance coverage \(\alpha_i/L_i\) (which is the expected payment in case of a loss) for expected unit indemnity payment \(i = 2.5\) and fixed complexity \(\varepsilon = 0.1 \times i = 0.25\), implying a relative premium loading on the actuarially fair price of \(\frac{1-p}{p} = 1/3\).

\((\varepsilon > 0)\), individuals face uncertainty about the actual indemnity payment and expect to receive either \(\alpha(i + \varepsilon)\) or \(\alpha(i - \varepsilon)\) in case the loss occurs. Given a fixed insurance coverage \(\alpha\), higher contract complexity thus implies that individuals face higher uncertainty.

Upon the purchase of \(\alpha\) units of insurance, an individual’s expected utility is given by

\[
EU(\alpha, \varepsilon, i) = p\mathbb{E}[u(w_0 - L + \alpha(i + \varepsilon - 1))] + (1 - p)u(w_0 - \alpha)
\]

\[
= \frac{p}{2} \left( u(w_0 - L + \alpha(i + \varepsilon - 1)) + u(w_0 - L + \alpha(i - \varepsilon - 1)) \right) + (1 - p)u(w_0 - \alpha),
\]

In the following, we denote the state-dependent utilities by \(u_x = u(w_x), u'_x = u'(w_x), u''_x = u''(w_x), u'''_x = u'''(w_x)\), where \(x \in \{1; 1; -; 1; +; 2\}\).

Without contract complexity, our model collapses into the standard model for insurance demand (Mossin (1968), Doherty (1975)). The first-order condition (FOC) then equals

\[
(i - 1)u'_1 = \frac{1-p}{p}u'_2,
\]

and full insurance \((\alpha i = L)\) is optimal if the premium is perceived as actuarially fair, i.e., if \(1 = pi\), implying \(i - 1 = \frac{1-p}{p}\) and thus \(u'_1 = u'_2\) by the FOC. This standard result is often referred to as Mossin’s Theorem. Partial insurance \((\alpha i < L)\) is optimal with a positive proportional premium loading, i.e., if \(pi < 1\), implying \(i - 1 < \frac{1-p}{p}\) and thus \(u'_1 > u'_2\).
Insurance demand changes with the introduction of contract complexity: If \( \varepsilon > 0 \), the insurance contract becomes risky itself, increasing an individual’s risk in the loss state upon purchasing insurance (see Figure 2). With contract complexity, the FOC does not only depend on marginal utility in the loss and no-loss states, but also on differential marginal utility within the loss state:

\[
(i - 1)\mathbb{E}[u'_1] - \varepsilon \frac{u'_{1,-} - u'_{1,+}}{2} = \frac{1 - p}{p} u'_2. \tag{3}
\]

Larger contract complexity does not affect marginal utility in the no-loss state \( u'_2 \) where no indemnity is paid. Instead, complexity raises (II) the differential marginal utility in the loss state, \( u'_{1,-} - u'_{1,+} \), since \( u''(\cdot) < 0 \), reflecting that insurance is less valuable with higher contract complexity. It also raises (I) the expected marginal utility in the loss state if marginal utility is convex. Since \( u'_2 \) is increasing with insurance coverage, contract complexity thus results in a trade-off between (I) more and (II) less insurance coverage to reduce (I) risk across the loss and no-loss state and (II) risk within the loss state. The ultimate effect depends on the convexity of marginal utility, which relates to third-order risk preferences, namely prudence. As a result, introducing contract complexity implies that Mossin’s Theorem may not hold any more.

The concept of prudence is introduced by Kimball (1990): Individuals are prudent if the third derivative of their utility function is positive, \( u''''(\cdot) > 0 \). Eeckhoudt et al. (1995) characterize prudent agents as those who prefer to attach a mean-preserving increase in risk to the good instead of to the bad states of the world. In line with the rationale of precautionary savings developed by Rothschild and Stiglitz (1971) and Kimball (1990), prudence might have two effects on insurance demand: On the one hand, risky indemnity payments make insurance less effective in mitigating overall risk, which might reduce insurance demand. On the other hand, individuals might insure more as a response to the increased risk in the loss state as a means to increase wealth in the worst possible state. The final effect depends on the degree of prudence as well as the level of contract complexity:

Lemma 3.1 (Precautionary insurance).

1. If individuals are not prudent \( (u''''(\cdot) \leq 0) \), insurance demand decreases with the level of contract complexity \( \varepsilon \).
For any $\varepsilon < i - 1$, insurance demand increases with $\varepsilon$ if individuals are sufficiently prudent such that

$$-\frac{\bar{u}'''}{\bar{u}'} > 1 - \frac{\alpha \varepsilon u''_{1,-} + u''_{1,1}}{\alpha (i - 1)},$$

(4)

where $\bar{u}''' = \frac{u''_{1,-} - u''_{1,1}}{w_{1,-} - w_{1,1}}$ and $\bar{u}'' = \frac{u''_{1,-} - u''_{1,1}}{w_{1,-} - w_{1,1}}$. If $\varepsilon \geq i - 1$ or individuals are not sufficiently prudent, insurance demand decreases with $\varepsilon$.

Kimball (1990) defines the state-dependent coefficient of absolute prudence by $PR = -u'''/w'$. We find that precautionary insurance is driven by the average slope and curvature of marginal utility in the loss state, $\bar{u}''$ and $\bar{u}'''$, respectively. A larger coefficient of absolute prudence $PR$ for $w \in [w_{1,-}, w_{1,1}]$ implies that also $-\bar{u}'''/\bar{u}'$ is larger and thus prudence indeed drives precautionary insurance.

The lemma implies that individuals’ marginal rate of substitution increases with contract complexity if individuals are sufficiently prudent and $\varepsilon < i - 1$, and vice versa. Therefore, in the presence of small contract complexity and sufficiently prudent individuals, willingness-to-pay for insurance is increasing with contract complexity. In this case, less financially literate individuals will buy more insurance coverage in competitive equilibrium (under symmetric information) if contract costs exclusively depend on insurance coverage.

**Corollary 3.1.** If individuals are sufficiently prudent and $\varepsilon < i - 1$, the marginal rate of substitution along indifference curves in coverage-price space increases with $\varepsilon$ at any contract-premium pair $(\alpha, P)$.

To provide an illustration of above findings, assume that individuals maximize exponential utility with constant absolute risk aversion. Exponential utility allows for a straightforward assessment of individuals’ degree of prudence since then the coefficient of absolute risk aversion $ARA$ equals the coefficient of absolute prudence$^{14}$ As illustrated in Figure 3, we show

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$^{13}$Note that a larger degree of prudence also changes the shape of the utility function and therefore the equilibrium allocation. Condition \[4\] needs to hold in equilibrium.

$^{14}$Calibrating $ARA$ for exponential utility is also complicated by the fact that it reflects both risk aversion and prudence. To highlight the effects of prudence, we sometimes consider a value of $ARA$ that seems large compared to experimental evidence for risk aversion (e.g., by Harrison and Rutström (2008) and Holt and Laury (2002)), such as $ARA = 0.2$. However, the calibration is consistent with empirical estimates for prudence. For example, in Ebert and Wiesen (2014)’s experiment to elicit prudence, the average individual behaves roughly consistently to $ARA = 0.18$ and $RRA = 2$. 

The existence of two opposing effects of contract complexity: On one hand, an increase in complexity reduces optimal insurance coverage with low prudence, as illustrated in Figure 3 (a). Hence, a relatively imprudent individual is not willing to accept additional overall risk resulting from more complex insurance, which makes market insurance less attractive. On the other hand, contract complexity is positively related to insurance demand if prudence is high and complexity is low, which is the situation in Proposition 3.1 (2) and Figure 3 (b). Following Fei and Schlesinger (2008), we call this effect precautionary insurance. Precautionary insurance occurs when individuals prepare for an increase in uncertainty about indemnity payments by increasing wealth in the worst state $w_{1,-}$ via increasing insurance coverage. This results from the marginal utility of insurance in the loss state, $u'_1(i - \tilde{\vartheta} - 1)$, being convex in the mean-zero complexity risk $\tilde{\vartheta}$. Then, the marginal benefit of insurance is increasing with the variability of $\tilde{\vartheta}$, resulting in larger demand for insurance.

If, however, the level of contract complexity is larger than the net payout of insurance, $\varepsilon > i - 1$, wealth in the worst possible state $w_{1,-}$ is decreasing with insurance coverage. Therefore, individuals cannot raise wealth in $w_{1,-}$ to prepare for uncertain indemnity by increasing insurance coverage. As a result, insurance demand unambiguously decreases with contract complexity if $\varepsilon > i - 1$, as illustrated in Figure 3 and proven in Lemma 3.1 (2). Therefore,

\[ dw_{1,-}/d\alpha = i - \varepsilon - 1 < 0 \text{ if } \varepsilon > i - 1. \]

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Figure 3: Optimal insurance coverage with respect to changes in complexity.

The figures depict the optimal insurance coverage ($I^*$) relative to the loss size ($L$) for changes in the level of complexity ($\varepsilon$) relative to the expected indemnity payment ($i$). The individual with initial endowment $w_0 = 100$ maximizes CARA utility with the coefficient of absolute risk aversion $ARA$ for a loss $L = 50$ that occurs with probability $p = 0.3$ and expected insurance unit indemnity payment $i = 2.5$, which implies a relative premium loading on the actuarially fair price of $1 - \frac{pi}{pi} = 1/3$. 

(a) $ARA = 0.05$.  
(b) $ARA = 0.2$. 

Note that $dw_{1,-}/d\alpha = i - \varepsilon - 1 < 0$ if $\varepsilon > i - 1$. 

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Complexity ($\epsilon/i$) Optimal States of Wealth ($w/w_0$)

\[ w_2 \]
\[ w_{1,+} \text{ (high)} \]
\[ w_{1,-} \text{ (low)} \]

(a) Optimal states of wealth ($ARA = 0.05$).

(b) Optimal states of wealth ($ARA = 0.2$).

Figure 4: Optimal states of wealth with respect to changes in complexity.
The figures depict individuals’ wealth (relative to the wealth endowment $w_0$) conditional on optimal insurance coverage for changes in the level of complexity ($\epsilon$) relative to the expected indemnity payment ($i$). Individual with initial endowment $w_0 = 100$ maximize CARA utility with a coefficient of absolute risk aversion $\gamma = 0.2$ for a loss $L = 50$ that occurs with probability $p = 0.3$. The expected indemnity per unit paid for insurance is $i = 2.5$ which implies a relative premium loading on the actuarially fair price equals $\frac{1-p_i}{p_i} = 1/3$. The vertical line in Figure (b) corresponds to $\epsilon = i - 1$.

we find that precautionary insurance does not only depend on the level of prudence but on the level of contract complexity itself, as well. This finding in particular distinguishes our study from models with insurance-independent background risk, where precautionary insurance results from an increase in insurance-independent risk for prudent individuals (e.g., Eeckhoudt and Kimball (1992), Gollier (1996), Fei and Schlesinger (2008)).

In Figure 4 we show the optimal states of wealth associated with the optimal insurance coverage from Figure 3. With a relatively low degree of prudence, individuals reduce insurance coverage to maintain a relatively small risk within the loss state, as Figure 4 (a) illustrates. In contrast, for a more prudent individual in Figure 4 (b), precautionary insurance amplifies the dispersion between the two possible loss states for $\epsilon < i - 1$, while this effect reverses for $\epsilon > i - 1$.

At the turning point, $\epsilon = i - 1$, contract complexity offsets the net insurance payout: in this case, wealth in the least favorable (loss) state, $w_{1,-}$, is independent of insurance coverage, since $w_{1,-} = w_0 - L + \alpha(i - 1 - \epsilon) = w_0 - L$. Thus, optimal insurance coverage is determined only by the trade-off between a large indemnity payment in $w_{1,+}$ and suffering no loss in $w_2$. This reduces the individual’s optimization problem to a two-state problem, analogous to the well-known binary insurance model (Doherty (1975)). Since then individuals cannot change
wealth in the worst loss state $w_{1,-}$, decisions are driven by risk aversion only, and partial insurance coverage becomes optimal for $\varepsilon = i - 1$:

**Corollary 3.2** ($\varepsilon = i - 1$). Assume that $\varepsilon = i - 1$. If insurance is perceived as actuarially fair ($i = 1/p$), optimal insurance coverage is determined by $\alpha^* = \frac{p}{2-p}L$ and results in an average indemnity payment of $\alpha^* i = L/(2-p) < L$. If insurance includes a subjective loading ($i < 1/p$), partial insurance is also optimal ($\alpha^* < L/i$).

### 3.2 Overinsurance

As shown in the previous section, prudence is a motive for precautionary insurance at small levels of contract complexity. We show that precautionary insurance can incentivize individuals to demand an average indemnity payment that exceeds the actual loss, $\alpha i > L$, which we refer to as overinsurance. Overinsurance occurs if individuals are sufficiently prudent:

**Proposition 3.1.** Let $i > 1$ and $p \in (0,1)$. If prudence is sufficiently large (at optimal insurance coverage) such that

\[
-\bar{u}'''' \bar{u}'' > \frac{1}{2\alpha(i-1)} \left(1 + \frac{1-pi}{\alpha \varepsilon^2 p} \left( -\frac{u'\left(\mathbb{E}[w_1]\right)}{\bar{u}''} \right) \right),
\]

then individuals demand overinsurance ($\alpha^* i > L$), where $\bar{u}'''' = \frac{u''''_{1,-} - u''''_{1,+}}{w_{1,-} - w_{1,+}}$ and $\bar{u}'' = \frac{u''_{1,-} - u''_{1,+}}{w_{1,-} - w_{1,+}}$.

If contracts are perceived to include a proportional loading ($pi < 1$), the threshold for the average degree of prudence $-\bar{u}'''' \bar{u}''$ is increasing with $-\frac{u'\left(\mathbb{E}[w_1]\right)}{\bar{u}''}$, which reflects individuals’ risk aversion. Stronger risk aversion reduces the minimum degree of prudence to result in overinsurance. The intuition is that more risk averse individuals exhibit a relatively higher willingness-to-pay for insurance and, thus, more easily demand overinsurance in the presence of complex contracts.

If insurance is perceived as actuarially fair, individuals already demand full insurance ($\alpha i = L$) in the case without contract complexity, i.e., for $\varepsilon = 0$. In this case, the threshold in (5) is independent from risk aversion and individuals demand overinsurance for any small positive level of contract complexity if they are sufficiently prudent:

**Corollary 3.3.** If insurance is perceived as actuarially fair ($i = 1/p$), for any $\varepsilon \in (0, i - 1)$ individuals demand overinsurance if

\[
-\bar{u}'''' \bar{u}'' > \frac{1}{2\alpha(i-1)}
\]
This result does not necessarily imply that, given individuals are sufficiently prudent, an insurance market equilibrium includes overinsurance. Instead, the result only implies that individuals demand overinsurance. If overinsurance is not offered by insurers, individuals demand the highest possible coverage up to the optimal level since marginal expected utility is monotonically decreasing in insurance coverage (see the proof of Lemma 3.1).

In practice, insurance companies usually do not offer overinsurance due to the principle of indemnity. This principle states that an indemnity payment should only replace the actual loss amount, thereby putting the insured back financially into his or her pre-loss situation. This is common law in the U.S. and in many European countries (Pinse Masons (2008)). It is, however, noteworthy that overinsurance may still result from differences in the insured’s and the insurer’s assessment of the loss. For example, one may think of new-for-old-insurance (reinstatement) policies or fire insurance policies where the indemnity can differ from the actual present value of what has been lost, since indemnity payments are fixed before the loss occurs. For example, U.S. health insurers typically pay a fixed rate per diem for hospital stays, regardless of the actual costs of treatments (Reinhardt (2006)). Similarly, automobile insurance policies typically include the possibility to receive a fixed indemnity payment $I$ instead of the insurer directly paying the repair costs. Thus, if one is able to repair damages for less than $I$ or, more generally, if an individual’s disutility from having a damaged car is smaller than receiving $I$, the individual is - from her own perspective - overinsured.

4 Cost of transparency and equilibrium

4.1 Contract complexity in a competitive equilibrium

Risk averse individuals prefer contracts without (experienced) complexity, since

\[
\frac{\partial EU}{\partial \varepsilon} = \alpha p \frac{u_1'}{2} - u_{1-}' < 0
\]

\[16\]

Thus, if insurers offer contracts with coverage \( \alpha \in C \subseteq \mathbb{R}_+ \) with \( \max \{ C \} < \alpha^* \), individuals purchase \( \max \{ \alpha \in C : \alpha \leq \alpha^* \} \), where \( \alpha^* \) is the optimal coverage resulting from maximizing expected utility (1) for \( \alpha \in \mathbb{R}_+ \).

\[17\]

Special treatments may however be excluded from fixed per diem rating.
for all $\alpha > 0$. In this section, we address the question under which circumstances contract complexity nevertheless may exist in equilibrium. We show that a positive level of contract complexity can occur in equilibrium if firms face transparency costs, i.e., if it is costly for firms to reduce contract complexity. Such transparency costs may arise, e.g., from preparing additional explanatory materials (such as key information documents), offering additional advice through brokers or service centers, or assessing whether the contract’s terms and conditions can be simplified. New regulatory changes in the European Union make some of these measures mandatory for member states (Hofmann et al. (2018)).

In our model, individuals experience the level of contract complexity $\varepsilon$. In this section, we assume that experienced complexity $\varepsilon$ consists of two parts: the contract’s actual complexity $\nu$ and individuals’ financial illiteracy (i.e., unsophistication). Then, the experienced contract complexity is $\varepsilon = \beta \nu$. Firms decide upon the level of actual contract complexity $\nu$. $\beta$ is exogenous and reflects the level of individuals’ financial illiteracy: the larger $\beta$, the more financially illiterate are individuals. If $\beta = 0$, individuals do not experience any contract complexity, i.e., understand any contract regardless of its complexity $\nu$.

We consider a market with free-entry and homogeneous risk-neutral firms whose profits are subject to transparency costs $\kappa = \kappa(\nu) > 0$ with $\kappa' < 0$ and $\kappa'' > 0$. Marginally reducing the level of complexity (and thus increasing transparency) costs $-\kappa'$. Expected firm profit is given by

$$\Gamma(i, \nu) = \alpha (1 - pi) - \kappa(\nu).$$

(8)

Firms compete over payout $i$ and contract complexity $\nu$, and offer all (co-insurance) contracts with expected indemnity payment $\alpha i, \alpha > 0$. Individuals choose optimal insurance coverage $\alpha$ among the contracts offered. Since we assume that financial illiteracy is primarily a risk effect but not necessarily a bias toward actuarial fairness, we assume that individuals’ expectation about the indemnity payment is unbiased in the sense that both firms and individuals expect the same per-unit payout $\$i$ (upon a loss), on average.\(^{18}\) The equilibrium allocation maximizes

\(^{18}\) Nonetheless, it is straightforward to extend our model to include a bias, e.g., that firms pay $i$ but individuals expect it to be $(1 + \lambda)i$, on average.
individuals’ expected utility subject to a non-negative expected profit constraint,

\[
\max_{\alpha \geq 0, \nu \geq 0, i \geq 0} EU(\alpha, \beta \nu, i) \tag{9}
\]

subject to \( \Gamma(i, \nu) \geq 0. \tag{10} \)

Both the objective (9) and the constraint (10) are concave in \((\alpha, \nu, i)\). Therefore, equilibrium is unique if it exists (and it exists on any closed interval due to continuity). Since expected profit is strictly decreasing in payout \(i\) and increasing in complexity \(\nu\), firms exactly break even in equilibrium.

Contracts break-even if \(\alpha^*(1 - p_i) - \kappa = 0\), where \(\alpha^*\) maximizes individuals’ expected utility:

\[
\alpha^* = \arg \max_{\alpha \geq 0} EU(\alpha, \beta \nu, i) \tag{11}
\]

\[
= \arg \max_{\alpha \geq 0} p u(w_0 - L + \alpha(i + \beta \nu - 1)) \left( u(w_0 - L + \alpha(i - \beta \nu - 1)) \right) + (1 - p)u(w_0 - \alpha). \tag{12}
\]

To simplify the illustration in the following, we will consider the equilibrium in \((\varepsilon, i)\)-space by computing zero-profit curves and indifference curves based on the optimal insurance demand \(\alpha^*\) for each \((\varepsilon, i)\)-pair. The slope of the zero-profit curve is then

\[
\left. \frac{di}{d\varepsilon} \right|_{\Gamma = 0} = \frac{\frac{\partial \alpha^*}{\partial \varepsilon}(1 - p_i) - \frac{1}{\beta} \kappa'(\varepsilon/\beta)}{p\alpha - \frac{\partial \alpha^*}{\partial i}(1 - p_i)}. \tag{12}
\]

Since \(\kappa' < 0\) and \(p_i < 1\) for \(\Gamma = 0\) and \(\kappa > 0\), the zero-profit curve is upward-sloping if transparency costs \(\kappa\) are sufficiently small to result in a small price loading \(1 - p_i\) at \(\Gamma = 0\) (or if insurance demand is sufficiently inelastic in the payout \(i\) and sufficiently inelastic or increasing in complexity \(\varepsilon\)). Then, a reduction in complexity \(\varepsilon\) (i.e., an increase in transparency) is offset by a reduction in the payout \(i\). If transparency costs \(\kappa\) are sufficiently convex in \(\varepsilon\), i.e., \(\kappa'' > 0\) sufficiently large, the zero-profit curve is also concave.

Indifference curves \(\{\varepsilon, i\}|_{EU=EU(\alpha^*, \varepsilon, i)}\) depict all pairs of experienced contract complexity \(\varepsilon = \beta \nu\) (assuming that financial illiteracy \(\beta > 0\)) and expected indemnity payment \(i\) that result in the same level of expected utility given the respectively optimal coverage \(\alpha^*\):

\[
\left. \frac{di}{d\varepsilon} \right|_{EU} = -\left. \frac{\partial EU}{\partial \varepsilon} \right|_{EU} / \left. \frac{\partial EU}{\partial i} \right|_{EU} = \frac{p(\alpha^* + \frac{\partial \alpha^*}{\partial \varepsilon} \varepsilon) u'_1 + \frac{\partial \alpha^*}{\partial i} (1 - p) - \frac{\partial \alpha^*}{\partial i} (1 - p)i u'_2}{p(\alpha^* + \frac{\partial \alpha^*}{\partial i} i) u'_1 + \frac{\partial \alpha^*}{\partial \varepsilon} (1 - p) u'_2}. \tag{13}
\]
Since an increase in experienced complexity $\varepsilon$ unambiguously reduces expected utility for all $\alpha > 0$, while an increase in expected payout $i$ unambiguously increases expected utility for all $\alpha > 0$, indifference curves are increasing, $\frac{di}{d\varepsilon} \bigg|_{EU} > 0$. The utility-gain from higher expected indemnity $i$ offsets the utility-loss from higher complexity $\varepsilon$.

Figure 5 depicts an illustrative example. Below and on the zero-profit curve, contracts make non-negative expected profit, and vice versa. The zero-profit curve is upward sloping since a higher contract complexity reduces transparency costs, enabling insurers to offer a higher indemnity payment. It is concave since an increase in complexity reduces marginal transparency costs.

Indifference curves are increasing with complexity, since the utility gain from a higher indemnity payment offsets the disutility from higher complexity. Indifference curves are also convex, reflecting that it becomes increasingly more difficult to offset complexity by increasing the expected indemnity payment: intuitively, differential marginal utility in the loss state in (13) increases with complexity, requiring a higher compensation to offset a marginal increase in complexity. A North-West shift of indifference curves reflects an increase in expected utility. In equilibrium, indifference curve and zero-profit curve are tangential.

Figure 5: Break-even line (straight), indifference curves (dotted and dashed), and equilibrium contract (dot).

The zero-profit curve depicts all $(\varepsilon, i)$ pairs of experienced complexity and expected indemnity with zero expected profit given optimal coverage $\alpha^*$, respectively. An indifference curve depicts all $(\varepsilon, i)$ combinations that result in the same level of expected utility. Individuals have CARA utility with constant absolute risk aversion $ARA = 0.02$ for an initial wealth $w_0 = 100$, loss $L = 50$, and loss probability $p = 0.3$. Transparency cost are $k(\nu) = k(\min(\nu - \nu_0, 0))^2$ with $\nu_0 = 1/p$ and (a) $k = 0.1$ and (b) $k = 0.3$. $k/p^2$ are the cost to entirely remove contract complexity.
As is intuitive from Figure 5, complexity maximizes expected utility among contracts on the zero-profit curve. Expected utility along the zero-profit curve is

\[ EU = \frac{p}{2} \left( u \left( w_0 - L + \frac{\alpha - \kappa}{p} + \alpha(\varepsilon - 1) \right) + u \left( w_0 - L + \frac{\alpha - \kappa}{p} - \alpha(\varepsilon + 1) \right) \right) \]

\[ + (1 - p)u(w_0 - \alpha), \quad (14) \]

where experienced contract complexity is \( \varepsilon = \beta \nu \). In equilibrium, contract complexity thus satisfies the first-order condition

\[ \frac{\partial EU}{\partial \varepsilon} = \frac{p}{2} \left( u_1^\prime(-\kappa'/p + \alpha) + u_1^\prime(-\kappa'/p - \alpha) \right) = 0 \]

\[ \iff \kappa' = -p\alpha \frac{(u_1^\prime - u_1^\prime)/2}{\mathbb{E}[u_1^\prime]}. \quad (15) \]

The right-hand-side of Equation (16) is negative if \( \alpha > 0 \) and decreasing with individuals’ risk aversion. Therefore, an inner solution (\( \nu/\beta > 0 \)) exists only if \( \beta > 0 \) and transparency costs are decreasing with complexity (and thus increasing with transparency): \( \kappa' < 0 \). Otherwise, \( \varepsilon = 0 \) and thus \( \nu = 0 \) is the optimal solution, as \( \frac{\partial EU}{\partial \varepsilon} < 0 \) for all \( \varepsilon, \alpha > 0 \).

Assume that \( \beta > 0 \). If an interior solution for \( \varepsilon \) exists, it is an expected utility maximum since

\[ \frac{\partial^2 EU}{\partial \varepsilon^2} = \frac{p}{2} \left( u_1^\prime(-\kappa'/p + \alpha)^2 + u_1^\prime(-\kappa'/p - \alpha)^2 \right) - \kappa''\mathbb{E}[u_1^\prime] < 0. \quad (17) \]

The interior solution is positive if \( \varepsilon = (\kappa')^{-1} \left( p\alpha \frac{(u_1^\prime - u_1^\prime)/2}{\mathbb{E}[u_1^\prime]} \right) > 0 \). Hence, a positive level of actual contract complexity \( \nu = \varepsilon/\beta \) is acceptable if marginal transparency costs \( |\kappa'| \) are sufficiently large and fast increasing.

For example, consider \( \kappa \) to be quadratic with a cost-minimum level of complexity \( \nu_0 \), such that \( \kappa = k(\nu - \nu_0)^2 \)

\[ 19 \]

Then, the equilibrium level of contract complexity satisfies

\[ \nu = \nu_0 - p\alpha \frac{(u_1^\prime - u_1^\prime)/2}{2\beta k\mathbb{E}[u_1^\prime]} \]

\[ \quad (18) \]

and is positive if (a) transparency costs or (b) cost-minimizing contract complexity \( \nu_0 \) are sufficiently large, and (c) loss probability \( p \) sufficiently small, given positive insurance coverage \( \alpha > 0 \). With this particular transparency cost function, high marginal costs for deviating from

\[ 19 \text{We only consider } \nu \leq \nu_0 \text{ in order to have transparency costs decreasing with complexity.} \]
the cost-minimum contract complexity \( \nu_0 \) decrease the expected indemnity payment. The smaller the coverage \( \alpha \) and the loss probability \( p \), the larger is the reduction in the expected indemnity payment \( i \) upon a marginal increase in transparency costs \( \kappa \). As a result, in equilibrium individuals accept a positive level of contract complexity in exchange for a higher payout. The following proposition summarizes our findings:

**Proposition 4.1.** Assume that transparency costs are convex and decreasing in contract complexity, \( \kappa' < 0, \kappa'' > 0 \). Then, a positive level of contract complexity \( \nu > 0 \) exists in equilibrium particularly if \( |\kappa'| \) is sufficiently large and \( p \) sufficiently small, given positive insurance demand.

### 4.2 Welfare and the financial illiteracy premium

We extend our analysis to estimate the welfare cost of financial illiteracy. For this purpose, we compare different levels of \( \beta \), reflecting different levels of financial literacy. In the most extreme cases, if \( \beta = 1 \), contract complexity is fully passed on to individuals, while individuals with \( \beta = 0 \) are perfectly financially literate, not experiencing contract complexity at all. We assume that an unique minimum \( \nu_0 \) for transparency costs exists, \( \kappa'(\nu_0) = 0 \) and \( \kappa''(\nu_0) > 0 \). For simplicity and without loss of generality, we assume that \( \kappa(\nu_0) = 0 \). For example, \( \nu_0 \) might correspond to a benchmark contract that is available to firms without additional costs.

Since indifference curves in \((\varepsilon, i)\)-space do not depend on actual contract complexity \( \nu \) but experienced contract complexity \( \varepsilon = \beta \nu \), a change in \( \beta \) does not alter indifference curves but zero-profit curves via the actual contract complexity \( \nu = \varepsilon / \beta \): a lower \( \beta \) increases the actual contract complexity to break even for a given \( \varepsilon \). For given \( \varepsilon \) and \( \nu < \nu_0 \), insurers can offer a higher indemnity \( i \) for lower \( \beta \) to break even. Figure 6 illustrates this effect by an upward shift of the zero-profit curve for small \( \varepsilon \). If, however, \( \varepsilon > \beta \nu_0 \), the implied actual complexity is larger than cost-minimum complexity, \( \nu = \varepsilon / \beta > \nu_0 \). Therefore, transparency cost increase again with higher contract complexity, resulting in a decreasing zero-profit curve for high \( \varepsilon \). Due to the upward shift of the zero-profit curve for small \( \varepsilon \), individuals attain a higher expected utility in equilibrium with low \( \beta \) (point B) than with high \( \beta \) (point A).\(^{21}\)

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\(^{20}\)In particular, along zero-profit curves the marginal rate of substitution between indemnity \( i \) and transparency costs \( \kappa \) is \( \frac{di}{d\kappa} = -\frac{\alpha p}{(\alpha p)^{-1}} \).

\(^{21}\)Note that studying indifference curves in \((\varepsilon, i)\)-space allows us to compare expected utility for different levels of financial literacy \( \beta \) since these only affect zero-profit curves.
Figure 6: Break-even lines (straight), indifference curves (dotted and dashed), and optimal contracts (dots). Point A corresponds to equilibrium with $\beta = 0$, point B to equilibrium with $\beta = 0.5$.

The zero-profit curve depicts all $(\varepsilon, i)$ pairs of experienced complexity and expected indemnity with zero expected profit given optimal coverage $\alpha^*$, respectively. An indifference curve depicts all $(\varepsilon, i)$ combinations that result in the same level of expected utility. Individuals maximize CARA utility with constant absolute risk aversion $ARA = 0.02$ for an initial wealth $w_0 = 100$, loss $L = 50$, and loss probability $p = 0.3$. Transparency cost are $k(\nu) = k(\nu - \nu_0)^2$ with $\nu_0 = 1/p$ and (a) $k = 0.1$ and (b) $k = 0.3$. $k/p^2$ are the cost to entirely remove contract complexity.

In the following, we focus on the welfare-loss due to financial illiteracy which is reflected by the differential expected utility in equilibrium with financially literate ($\beta = 0$) and illiterate ($\beta = 1$) individuals. If $\beta = 0$, individuals do not experience disutility from contract complexity and thus, in a competitive equilibrium, firms choose the level of contract complexity that minimizes transparency cost. Thus, $\nu^* = \nu_0$ in equilibrium. Break-even indemnity payments then satisfy $i = 1/p$, i.e., are actuarially fair. Thus, for $\beta = 0$ the zero-profit curve in $(\varepsilon, i)$-space is flat with $i = 1/p$. Individuals maximize

$$EU|_{\beta=0} = pu(w_0 - L + \alpha(i - 1)) + (1 - p)u(w_0 - \alpha)$$

over coverage $\alpha$. It is well-known that the solution and equilibrium to this problem is full coverage, $\alpha^*i = L$ (e.g., see Doherty (1975)), such that expected utility in equilibrium is $EU^*|_{\beta=0} = u(w_0 - pL)$.

To compare welfare with and without financial illiteracy, we translate the welfare-loss from financial illiteracy into monetary cost as given by the financial illiteracy premium $C$ such that

$$u(w_0 - pL - C) = EU^*|_{\beta=1},$$

(20)
where $EU^*|_{\beta=1}$ is the expected utility in equilibrium with financially illiterate individuals, $\beta = 1$. $C$ is individuals’ maximum willingness-to-pay to completely remove financial illiteracy, reflecting the welfare cost of financial illiteracy. It also reflects the utilitarian social welfare cost of financial illiteracy in (fully competitive) markets with zero profits to risk-neutral firms. It is straightforward to show that $C > 0$ whenever the equilibrium with $\beta = 1$ entails a positive but small level of complexity $0 < c^* = \beta \nu^* < w_0 - L + \alpha^* (i - 1)$, and there is no price discount (i.e., $ip \leq 1$), since risk aversion then implies that

$$EU|_{\beta=0} = u(w_0 - pL) \geq pu(w_0 - L + \alpha^* (i - 1)) + (1 - p)u(w_0 - \alpha^*)$$

$$> pu(w_0 - L + \alpha^* (i + \beta \nu^* - 1)) + u(w_0 - L + \alpha^* (i - \beta \nu^* - 1))$$

$$+ (1 - p)u(w_0 - \alpha^*)$$

$$= EU^*|_{\beta=1}.$$  

(21)  

(22)  

(23)

In Figure 7, we examine the sensitivity of the illiteracy premium towards different key parameters of the model. We rely on exemplary parameters: Individuals have initial wealth $100$, maximize exponential utility with constant absolute risk aversion $ARA = 0.02$ and face a loss of $50$ that occurs with probability $30\%$. The implied coefficient of relative risk aversion is $RRA = 1.7$ for expected uninsured wealth, which is consistent with the degree of risk aversion revealed by subjects in Ebert and Wiesen (2014)’s experiment during tasks that elicit their degree of prudence. First, one should note that the illiteracy premium $C$ can be relatively large compared to initial wealth $w_0$: For a reasonable calibration, the illiteracy premium increases up to $3\%$ of initial wealth, which seems substantial. On the flip side, the illiteracy premium vanishes if (a) marginal transparency cost are small or (b+c) individuals are risk neutral. (a) If marginal transparency costs are zero, individuals accept a high complexity in equilibrium, approaching the optimal policy for financially literate individuals. (b+c) If individuals are risk neutral, they do not purchase insurance as the net present value is not positive. \(^{22}\)

We have shown that financially illiterate individuals accept a high level of contract complexity if marginal transparency costs $\kappa'$ are high. The higher the transparency cost, the lower is the expected indemnity payment for the insurer to break even, and thus, the smaller is insurance demand. Therefore, transparency cost increase the illiteracy premium, as Figure 7(a) shows.

\(^{22}\)More specifically, individuals are indifferent between purchasing insurance with initial complexity $\varepsilon_0$ at the actuarially fair price and purchasing no insurance. If we assume that they purchase insurance with initial complexity $\varepsilon_0$, our result does not change.
Figure 7: Sensitivity of the financial illiteracy premium towards changes in (a) marginal transparency costs \( k/p^2 \) scaled by initial wealth \( w_0 \), (b) the coefficient of absolute risk aversion \( ARA \) and prudence with CARA utility, and (c) the coefficient of absolute risk aversion \( ARA \) at average wealth \( w_0 - pL \) for quadratic utility.

In Figures (a) we maximize CARA utility with constant absolute risk aversion \( ARA = 0.02 \) which also equals the degree of absolute prudence, in (b) we maximize CARA utility with varying coefficient of absolute risk aversion \( ARA \), in (c) we maximize quadratic utility \( u(w) = aw - \gamma w^2 \) for \( \frac{a}{2\gamma} > \gamma \) such that \( u' > 0 \) for all attainable values. Initial wealth is \( w_0 = 100 \), the loss is \( L = 50 \), and the loss probability is \( p = 0.3 \). Transparency cost are given by \( \kappa(\nu) = k(\nu - \nu_0)^2 \) with \( \nu_0 = 1/p \) such that \( k/p^2 \) are the cost to entirely remove contract complexity. It is \( k = 0.3 \) in Figures (b), and (c). Note that \( ARA = 0.02 \) corresponds to \( RRA = 1.7 \) at wealth \( w_0 - pL = 85 \).

Thus, the welfare cost of financial illiteracy are higher if it is more costly for firms to deviate from existing levels of contract complexity.

We also show in Figure 7 (b) and (c) that the illiteracy premium is increasing with risk aversion. Intuitively, less risk averse illiterate individuals are less sensitive towards changes in contract complexity. Therefore, in equilibrium these individuals accept a higher level of contract complexity in exchange for a smaller price. Figures 7 (b) and (c) differ with respect to preferences: We use exponential utility (constant absolute risk aversion) in Figure 7(b) and
quadratic utility in Figure 7(c). Exponential utility is mostly standard in the literature but implies that we cannot alter risk aversion and prudence separately: The coefficient of relative risk aversion also determines prudence (Eeckhoudt and Schlesinger (1994)). Thus, in Figure 7(b) it is challenging to disentangle the effects of prudence and risk aversion. To overcome this issue, we compare the illiteracy premium of exponential utility to quadratic utility in Figure 7(c) where $u'''(\cdot) = 0$, i.e., individuals are not prudent for any level of absolute risk aversion $ARA$. We find that changes in risk aversion have a similar effect for quadratic utility in Figure 7(c) as for exponential utility in Figure 7(b). We conclude that $C$ is increasing in risk aversion and that it is not (only) prudence that drives $C$. This is intuitive since larger risk aversion implies a larger disutility from contract complexity, resulting in smaller levels of contract complexity and higher prices in equilibrium with illiterate individuals. This raises the illiteracy premium.

4.3 Policy implications

Making insurance contracts more understandable to consumers is an important challenge for insurance regulators worldwide. Generally, there are two main ways to reduce social costs of financial illiteracy: 1) Transparency requirements for insurance providers to reduce contract complexity, and 2) increasing financial literacy of consumers (e.g., via consumer education). In recent years, policymakers have undertaken substantial efforts in pursuing the first way by imposing regulatory transparency standards: The National Association of Insurance Commissioners founded the Transparency and Readability of Consumer Information (C) Working Group in 2010 in order to develop best practices for increasing transparency in the U.S. insurance market. Recently, the European Union implemented a standardized document to improve transparency in the EU in the form of the Insurance Product Information Document (IPID), which overviews all key features of an insurance contract (i.e., obligations of all parties, claims handling, and insurance coverages) in a ”standardized presentation format”[^23]. Yet, such transparency regulation requires insurers to implement costly additional measures to increase contract transparency (German Insurance Association (GDV) (2016)). Insurers are likely to recover these additional costs from consumers via loadings on insurance prices.

This study proposes a framework for assessing the welfare effects of financial illiteracy in competitive markets. We show that positive complexity exists as an equilibrium phenomenon under reasonable assumptions when individuals are financially illiterate. However, as a result from perfect competition, any deviation from the equilibrium level of contract complexity is welfare-decreasing, as the utility from smaller complexity does not offset the disutility from higher prices. Hence, transparency regulation that determines a fixed maximum level of complexity is welfare-decreasing in our framework, particularly if it very costly for firms to reduce complexity. In contrast, we find that financial illiteracy imposes welfare costs of 1% to 3% of wealth for a reasonable calibration, implying that an increase in financial literacy, e.g. via consumer education, unambiguously raises welfare as long as the associated costs do not exceed the financial illiteracy premium of 1-3% of the overall wealth endowment.

Nevertheless, one should not interpret our results as a pledge against transparency regulation. Instead, our analysis highlights that there is no space for welfare-increasing transparency regulation in friction-less markets in which insurers compete over the complexity of products and (financially illiterate) consumers costlessly choose among the products offered. Nonetheless, market frictions like search costs, an oligopolistic market structure of firms, or behavioral biases of consumers (that, e.g., let them favor products of well-known firms despite higher complexity) might still provide a rationale for transparency regulation. Therefore, we provide a starting point for future research to explore the equilibrium implications of (different forms) of transparency regulation in different market environments. Moreover, the insight that transparency regulation is not unambiguously welfare-increasing should motivate policymakers to provide precise descriptions on the market frictions that (transparency) regulations target.

5 Conclusion

This study shows that consumers’ degree of financial illiteracy has a profound impact on insurance market outcomes. In particular, insurance demand is different from standard findings. It strongly interacts with risk aversion and individual levels of prudence, which might both increase and decrease demand for insurance. We identify a threshold for prudence such that more prudent individuals demand higher levels of coverage upon an increase in contract complexity. We refer to this effect as precautionary insurance, which has been discussed in the literature.
The findings have important implications for studying individual decision-making under risk and the impact of financial literacy on market outcomes. Typically, under-insurance (i.e., payouts being smaller than losses) is interpreted as a sign for a low level of financial literacy (e.g., Quantum Market Research for the Insurance Council of Australia (2013), Fairer Finance (2018)). However, our results imply that financial illiteracy might as well result in excessive demand for insurance by prudent individuals who desire to raise the minimum payout in case of a loss. As a result, contract complexity may actually increase insurance demand.

When adding an equilibrium model to the analysis, contract complexity is endogenized. It is shown that contract complexity exists in equilibrium if contract transparency is costly for firms. We quantify the resulting welfare loss from financial illiteracy as the financial illiteracy premium, which amounts to 1 to 3% of the overall endowment. Our analyses demonstrate the differential effects of regulatory actions to reduce welfare losses from illiteracy, particularly minimum transparency standards vs. financial education. By imposing minimum transparency standards, the disutility from a price increase due to transparency standards might exceed the utility from smaller contract complexity, further reducing welfare in friction-less competitive markets. In such markets, measures to increase financial literacy, e.g. via financial education programs, are likely to reduce equilibrium prices and, thus, individuals benefit from both smaller prices and smaller financial illiteracy. As a consequence, policymakers should carefully assess the costs and benefits of the two approaches to address potential inefficiencies arising from consumer financial illiteracy.
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Appendix

Proof of Lemma 3.1

Proof.

(1) Assume that individuals are not prudent, i.e. \( u'''(\cdot) \leq 0 \). The FOC for optimal insurance coverage is

\[
\frac{\partial EU}{\partial \alpha} = \frac{p}{2} (u'_{1,+}(i + \varepsilon - 1) + u'_{1,-}(i - \varepsilon - 1)) - (1 - p)u'_2 = 0.
\] (5.24)

Accordingly, we arrive at the following second order condition:

\[
d^2EU
\frac{d\alpha}{d\alpha^2} = \frac{p}{2} (u''_{1,+}(i + \varepsilon - 1)^2 + u''_{1,-}(i - \varepsilon - 1)^2) + (1 - p)u''_2 < 0,
\] (5.25)

which is negative as \( u'' < 0 \), and thus the solution \( \alpha^* \) to (5.24) is unique. Optimal insurance coverage is decreasing with \( \varepsilon \) if the FOC is decreasing with \( \varepsilon \). This is the case if

\[
d^2EU\frac{d\alpha}{d\alpha d\varepsilon} = \frac{p}{2} (u''_{1,+}\alpha(i + \varepsilon - 1) - u''_{1,-}\alpha(i - \varepsilon - 1) + u'_{1,+} - u'_{1,-}) < 0,
\] (5.26)

where \( u'_{1,+} - u'_{1,-} < 0 \) due to risk aversion.

Let \( \varepsilon < i - 1 \). Then, it is \( 0 < i - \varepsilon - 1 < i + \varepsilon - 1 \) and thus

\[
u''_{1,+}\alpha(i + \varepsilon - 1) - u''_{1,-}\alpha(i - \varepsilon - 1) < 0
\] (5.27)

\[
\Leftrightarrow \frac{i + \varepsilon - 1}{i - \varepsilon - 1} > \frac{u''_{1,-}}{u''_{1,+}}.
\] (5.28)

\( u'''(\cdot) \leq 0 \) implies that \( u''_{1,-} \geq u''_{1,+} \Leftrightarrow \frac{u''_{1,-}}{u''_{1,+}} \leq 1 \). Since the LHS of (5.28) is larger than unity, (5.28) and thus (5.26) holds.

In the following, we do a case by case analysis depending on \( \varepsilon \): Let \( \varepsilon \geq i - 1 \). Then, it is \( i - \varepsilon - 1 \leq 0 < i + \varepsilon - 1 \) and thus \(-u''_{1,-}\alpha(i - \varepsilon - 1) \leq 0\) and \( u''_{1,+}\alpha(i + \varepsilon - 1) < 0\), implying (5.28) and thus (5.26). Therefore, (5.26) always holds if \( u''' \leq 0 \) and thus optimal insurance coverage is decreasing with contract complexity \( \varepsilon \).
(2) Assume that \( \varepsilon < i - 1 \) and let \( \mathbb{P}(\tilde{\vartheta} = \varepsilon) = \mathbb{P}(\tilde{\vartheta} = -\varepsilon) = 1/2 \). The proof aims at finding a boundary for the level of prudence such that (5.26) > 0. This can be rewritten as

\[
\begin{align*}
& u''_{1,+} \alpha(i + \varepsilon - 1) - u''_{1,-} \alpha(i - \varepsilon - 1) > -(u'_{1,+} - u'_{1,-}) \quad (5.29) \\
\iff & \alpha(i - 1) \left( u''_{1,+} - u''_{1,-} \right) + \alpha \varepsilon \left( u''_{1,+} + u''_{1,-} \right) > -(u'_{1,+} - u'_{1,-}) \quad (5.30) \\
\iff & \alpha(i - 1) \frac{u'_{1,+} - u'_{1,-}}{u'_{1,-} - u'_{1,+}} + \alpha \varepsilon \frac{u''_{1,+} + u''_{1,-}}{u'_{1,-} - u'_{1,+}} > 1 \quad (5.31) \\
\iff & -\frac{(u''_{1,-} - u''_{1,+})}{(w_{1,-} - w_{1,+})} > \frac{1 - \alpha \varepsilon u'_{1,-} - u'_{1,+}}{(w_{1,-} - w_{1,+})} \quad (5.32)
\end{align*}
\]

The LHS approximates the degree of absolute prudence, \( PR = -\frac{u''}{u'} \). The inequality therefore holds if the degree of absolute prudence is sufficiently large.

Assume that \( \varepsilon \geq i - 1 \). From (5.26), it follows that \( \frac{d\alpha^*}{d\varepsilon} < 0 \) if

\[
\frac{u''_{1,+}(i + \varepsilon - 1) - u''_{1,-}(i - \varepsilon - 1)}{\alpha} < \frac{u'_{1,-} - u'_{1,+}}{\alpha}.
\]

Since \( \varepsilon \geq 1 - P \), it is \( A > 0 \) and \( B \leq 0 \). Then, the LHS of (5.33) becomes negative and insurance demand is decreasing with complexity irrespective of the degree of prudence.

\[ \square \]

**Proof of Corollary 3.1:**

**Proof.** Fix \( i > 1 \) and let \( \varepsilon < i - 1 \). Individuals derive utility \( EU = p\mathbb{E}[u(w_0 - L - P + \alpha(i + \tilde{\vartheta}))] + (1 - p)w_0 - P \) from buying coverage \( \alpha \) at price \( P \). The marginal rate of substitution along an indifference curve in \( \alpha - P \) space is given by

\[
\frac{dP}{d\alpha} \bigg|_{EU=\text{const}} = \frac{p\mathbb{E}[u'_1(i + \tilde{\vartheta})]}{p\mathbb{E}[u'_1] + (1 - p)u'_2}. \quad (5.34)
\]

Analogously to Rothschild and Stiglitz (1971), the impact of an increase in risk of \( \tilde{\vartheta} \) (i.e., an increase in \( \varepsilon \)) on \( \mathbb{E}[u'_1] \) and \( \mathbb{E}[u'_1 \tilde{\vartheta}] \) depends on whether \( u'_1 \) and \( u'_1 \tilde{\vartheta} \) are convex or concave in \( \tilde{\vartheta} \). If both are convex, an increase in risk leads to an increase in \( \mathbb{E}[u'_1] \) and \( \mathbb{E}[u'_1 \tilde{\vartheta}] \). \( u'_1 \) is convex in \( \tilde{\vartheta} \) if \( u'' > 0 \) because \( \frac{\partial u'_1}{\partial \tilde{\vartheta}^2} = \alpha^2 u'' \). \( u'_1 \tilde{\vartheta} \) is convex in \( \tilde{\vartheta} \) if, and only if,

\[
\frac{\partial^2 u'_1 \tilde{\vartheta}}{\partial \tilde{\vartheta}^2} = u'' \tilde{\vartheta} \alpha^2 + 2u'' \alpha > 0,
\]

\[ 33 \]
which is equivalent to \(-\frac{u''}{u_1'} > \frac{2}{\alpha}\). Hence, if individuals are sufficiently prudent such that \(-\frac{u''}{u_1'} > \frac{2}{\alpha} \geq \frac{2}{\alpha}\) in equilibrium, an increase in contract complexity \(\varepsilon\) leads to an increase in \(\mathbb{E}[u_1']\). Because \(i \geq 1\), upon an increase in variability of \(\tilde{\theta}\), the increase in the numerator of (5.34), and particularly of \(\mathbb{E}[u_1']\), is at least as large as the increase in the denominator of \(\mathbb{E}[u_1']\). Therefore, for any contract \(\alpha\) and price \(P\) the marginal rate of substitution is increasing with \(\varepsilon\).

\[\square\]

**Proof of Corollary 3.2**

*Proof.* Assume that \(\varepsilon = i - 1\). Then, it is \(w_{1,-} = w_0 - L, w_{1,+} = w_0 - L + \alpha(i + \varepsilon - 1) = w_0 - L + 2\alpha(i - 1)\), and \(w_2 = w_0 - \alpha\). Optimal insurance coverage satisfies

\[
\frac{\partial EU}{\partial \alpha} = \frac{p}{2} u_{1,+}'(i - 1) - (1 - p)u_2' = 0
\]

\[\Leftrightarrow \quad \frac{u_{1,+}'}{u_2'} = \frac{1 - p}{i - 1} \quad (5.36)
\]

If insurance is (subjectively) actuarially fair, it is \(i = 1/p\), implying that \(i - 1 = \frac{1 - p}{p}\) and, thus, \(u_{1,+}' = u_2'\), which is equivalent to \(-L + 2\alpha \frac{1 - p}{p} = -\alpha \Leftrightarrow \alpha \frac{2 - p}{p} = L \Leftrightarrow \alpha = L/2 - p\) and results in an expected indemnity payment \(\alpha i = L/(2 - p)\).

If insurance includes a (subjective) premium loading, it is \(i < 1/p\), implying that \(i - 1 < \frac{1 - p}{p}\) and, thus, \(\frac{u_{1,+}'}{u_2'} > 1\) or equivalently \(w_{1,+} < w_2 \Leftrightarrow -L + 2\alpha(i - 1) < -\alpha \Leftrightarrow \alpha(1 + 2(i - 1)) = \alpha(2i - 1) < L \Leftrightarrow \alpha < \frac{L}{2i - 1} < \frac{L}{i} \) if \(i > 1\), which implies partial insurance.

\[\square\]

**Proof of Proposition 3.1**

*Proof.* Overinsurance occurs if wealth in the no-loss state is smaller than expected wealth in the loss state, \(w_2 < \mathbb{E}[w_1]\), or, equivalently, \(u'(w_2) > u'(\mathbb{E}[w_1])\). Using the first-order condition for insurance demand, overinsurance is optimal if

\[
u'(\mathbb{E}[w_1]) \quad \frac{p}{1 - p} \mathbb{E}[u'(w_1)](i - 1) + \frac{p}{1 - p} \frac{\varepsilon}{2} (u_{1,+}' - u_{1,-}')
\]

\[\Leftrightarrow - \frac{p}{1 - p} \frac{\varepsilon}{2} (u_{1,+}' - u_{1,-}') \quad \frac{p(i - 1)}{2(1 - p)} \left[ u_{1,+}' - u'(\mathbb{E}[w_1]) - (u'(\mathbb{E}[w_1]) - u_{1,-}') + 2u'(\mathbb{E}[w_1]) \right]
\]

\[- u'(\mathbb{E}[w_1]) \].

Define by \(\tilde{u}'' = \frac{u_{1,+}' - u_{1,-}'}{2\alpha \varepsilon} < 0\) the first order difference quotient of \(u'\), reflecting the average slope of \(u'(w)\) for \(w \in (w_{1,-}, w_{1,+})\), and by \(\hat{u}'' = \frac{u_{1,+}' - u'(\mathbb{E}[w_1]) - (u'(\mathbb{E}[w_1]) - u_{1,-}')}{4\alpha \varepsilon^2}\) the second
order difference quotient of \( u' \), reflecting the average curvature of \( u'(w) \) for \( w \in (w_1, ..., w_1, +) \) in equilibrium. Then, overinsurance is optimal if

\[
- \frac{p}{1-p} \varepsilon^2 \alpha \dddot{u} < \frac{2 \alpha^2 \varepsilon^2 p}{(1-p)} (i-1) \dddot{u} - \frac{1-p}{1-p} u'(\mathbb{E}[w_1])
\]

(5.40)

\[
\Leftrightarrow \frac{p}{1-p} \varepsilon^2 \alpha < \frac{2 \alpha^2 \varepsilon^2 p}{(1-p)} (i-1) \left( -\frac{\dddot{u}}{\dddot{u}} \right) - \frac{1-p}{1-p} \left( -\frac{u'(\mathbb{E}[w_1])}{\dddot{u}} \right)
\]

(5.41)

\[
\Leftrightarrow \varepsilon^2 \alpha + (1-pi) \left( -\frac{u'(\mathbb{E}[w_1])}{\dddot{u}} \right) < 2 \alpha^2 \varepsilon^2 p(i-1) \left( -\frac{\dddot{u}}{\dddot{u}} \right)
\]

(5.42)

\[
\Leftrightarrow \frac{1}{2\alpha(i-1)} + \frac{1-\alpha}{2\alpha \varepsilon^2 p(i-1)} \left( -\frac{u'(\mathbb{E}[w_1])}{\dddot{u}} \right) < -\frac{\dddot{u}}{\dddot{u}},
\]

(5.43)

\[
\Leftrightarrow \frac{1}{2\alpha(i-1)} \left( 1 + \frac{1-pi}{\alpha \varepsilon^2 p} \left( -\frac{u'(\mathbb{E}[w_1])}{\dddot{u}} \right) \right) < -\frac{\dddot{u}}{\dddot{u}},
\]

(5.44)

where \( -\frac{\dddot{u}}{\dddot{u}} \) approximates the degree of prudence and \( -\frac{u'(\mathbb{E}[w_1])}{\dddot{u}} \) the inverse of the degree of risk aversion.

Proof of Corollary \[3.3\]

Proof. Assume that \( i = 1/p \). Thus, individuals demand full insurance if \( \varepsilon = 0 \). We show in Lemma \[3.1\] that insurance demand is increasing with contract complexity for any \( \varepsilon \in (0, i-1) \) and sufficiently prudent individuals, \( \frac{d\alpha^*}{d\varepsilon} > 0 \). Thus, if individuals are sufficiently prudent, it is \( \alpha^*(\varepsilon)i \geq L \) for any \( \varepsilon \in (0, i-1) \), since otherwise \( \frac{d\alpha^*}{d\varepsilon} < 0 \) for some \( h \in (0, i-1) \). Hence, for any \( \varepsilon \in (0, i-1) \) there exists \( 0 < h < \varepsilon \) such that \( \alpha^*(h)i \geq L \) and \( \frac{d\alpha^*}{d\varepsilon}(k) > 0 \) for all \( k \in [h, \varepsilon) \). Thus, \( \alpha^*(\varepsilon)i > L \). \( \square \)