Insurance in the Age of Wisdom and Foolishness: A Tale of Healthy City

Abstract. We model the political economy for the provision of public long-term care services in an economy where the demand for long-term care services is jointly determined with the demand for long-term care insurance. We make use of Salop’s circular-city approach with randomly determined search costs and free entry in the supply of long-term care services. We show that long-term care services providers extract a rent that depends on the level of long-term care insurance that individuals purchased, and that this rent is proportional to the extend of services that are required.

JEL classification: G02, G12, C14.

Keywords: Longevity risk, Supply and demand of long-term care services and insurance; Salop’s circular city; Government programs; Informal care.
“It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to Heaven, we were all going direct the other way – in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.”

(Charles Dickens)

1 Introduction

One of the biggest sociological and financial challenges for OECD countries\(^1\) in the coming decades, aside from global warming and ecological changes, are demographic in nature. As OECD countries see their population growing older to soon reach a point where the ratio of workers to retirees approaches 2, one major challenge resides in the design and the financing of a system that will respond to the greater need for health services in those older ages, and in particular the need for a clearer including long-term care services. The provision of long-term care services is only one of many problems that OECD countries must face with respect to their growing elderly population. In contrast to medical care for the elderly and retirement planning for which great advances have been made, the great majority of OECD countries are still looking for the best way to provide and finance the need for long-term care services.

Long-term care is defined as the care for elderly individuals over a prolonged period of time. This care is provided in the form of support with activities of daily living (such as bathing, dressing, eating, getting in and out of bed, grooming, and continence) or with instrumental activities of daily living (which include preparing meals, cleaning, doing the laundry, taking medication, getting to places beyond walking distance, shopping, managing money, and using the telephone or the Internet). LTC is thus related to the loss of autonomy brought on by old age. It is important to distinguish upstream (acute care or rehabilitation) from downstream (help with activities of daily living) services since the former is generally taken care of by health professionals, whereas the latter is often provided by relatively unskilled workers and family members.

LTC should be distinguished from illness, disability, and handicap, which can affect younger individuals. Because needing LTC is not the same as having a disability, LTC insurance is not the same as disability insurance. Disability insurance is more targeted towards the working age population whereas LTC insurance is targeted towards the retired or soon-to-be-retired population.

Financing LTC services raises many challenges since LTC is becoming an increasingly important problem for all developed countries. According to OECD (2011), the population aged 80 and over is expected to

\(^1\)Help Wanted? - Providing and Paying for Long-Term Care. OECD Press.
represent 10% of the developed world’s population by 2050. That age bracket represented only 4% of the rich world’s total population in 2010. The over 80 age range is the fastest growing age group in the developed world. The fact that the proportion of the population that is elderly increases would not be a problem in itself if the up-and-coming population aged 80 in 2050 were as healthy as the population aged 70 in 2010. The challenge for the provision and the financing of LTC services is that the average number of years during which LTC services will be needed may actually increase if the population grows older but not healthier, or that the types of services needed, sought, and/or covered by the public system or private insurers in the future changes.

Some studies (see the references in Brown and Finkelstein 2008, 2009) argue that private LTC insurance contracts are expensive because of important loading factors. Brown and Finkelstein (2007) show, however, that loads on LTC insurance are not particularly high; at least not so high as to lead rich retirees to prefer using their private savings as a form of self-insurance rather than purchasing LTC insurance. Other studies (Sloan and Norton, 1997) point to the existence of important asymmetric information problems (both moral hazard and adverse selection) which induce insurers to restrict coverage.

Demand for LTC Services and Insurance

We first present in the next section the situation and the challenges that OECD countries with respect to the growing elderly population in possible need of LTC services and insurance. In Section 3, we present the model we propose to analyse the problem. We then examine supply side reactions on the LTC services market and the crowding-out by government programs. Finally we conclude.

2 Background on long-term care

2.1 The state of LTC services and insurance

Long-term care is a storm in waiting. Every OECD country is facing a rapidly aging population which is projected to be in more and more needs for long-term care services. For instance in Canada, the Conference Board, a non-partisan think tank, anticipates that the country will need an additional 199,000 long-term beds by 2035. Give that the number of beds available in 2018 is estimated to be 255,000, the Conference Coard anticipates a growth rate in the number of beds of 3.5% per year. In contrast, the average annual growth in the proportion of 65 year olds needing institutional care has been approximately 1% in the following set of OECD countries.

2.2 Supply of LTC facilities and LTC insurance characteristics

The following figure provides the proportion of the population receiving long-term care services in 2000 and 2013. We note that in most country, the proportion of the population receiving long-term care services has increased over these 13 years. On average, for the 21 OECD countries for which there are reliable value,
Figure 1: The change in the share of over 65 year olds using institutional care in selected OECD countries between 2003 and 2007 is calculated as the average share from 2006 and 2008 divided by the average share from 2002 and 2004 minus 1.

Despite the clear and present investment in long-term care facilities, private long-term care insurance is not very developed in any OECD country. The following Figure 3 shows that in OECD countries, the private LTC insurance market represents between 1% and 2% of total long-term care expenditures, with the great majority of countries having essentially no private long-term care insurance market to speak of. The case of Switzerland is interesting in the sense that although private expenditure on long-term care services represents almost 1.5% of GDP, less than 0.5% of this amount is actually financed by insurance companies. In the United States, 7% of all long-term care expenses are financed by insurance. This represents between 15% and 18% of all private long-term care expenditures in the country.

The private long-term care insurance market remains small despite the important social costs associated with dependency in the last years of life. One of those costs is the

### 2.3 Government programs

An important aspect of the LTC services market is the very large presence of government services which could be crowding-out private alternatives both in terms of having their own facilities and/or insurance schemes. Overall, OECD countries spend on average 1.5% of GDP on LTC services (see Figure 4), of which only 20% can be considered as private expenditures.

We observe in every OECD country — except Switzerland — that public expenditures in LTC services is
Figure 2: Proportion of the total population receiving long-term care services in selected OECD countries in 2000 and 2013 (source: OECD, 2015; Health at a Glance http://dx.doi.org/10.1787/health-data-en).

Figure 3: Private long-term care insurance market as a percentage of total LTC expenditures for selected OECD countries in 2008 (source: OECD, 2011; http://www.oecd.org/els/health-systems/47887332.pdf).
larger than private expenditures. We note, in particular, the 100% market share of public services when it comes to long-term care in Sweden, the Netherlands, Ireland, and France even though they all spend more in percentage of their GDP than the average OECD country. The good news, is that despite the demographic trend towards having older populations and the pressure it puts on health care cost at older ages, OECD countries have still been able to keep LTC expenditures (either public or private) at a relatively low levels.

This means that the perception on the insurance company side may be that there is very little demand for a product that would cover losses of at most $21,600 per year. Boyer et al. (2017) come up with a present value, at age 65, of the expected cost of long-term care services in Ontario of less than $20,000 ($13,000 in Quebec). They reach that conclusion assuming that

- half of the population aged 65 and over will require some form of nursing home;
- nursing home residents use the service for 5 years on average, and
- individuals will need a nursing home at age 80 (so 15 years later) on average.

Assuming there are fixed costs to selling LTC insurance contracts (that is, assuming a fixed insurance premium loading) of $10,000,\(^2\) it is quite possible that many individuals’ willingness-to-pay is smaller than the insurance industry’s break-even premium of $30,000. If this is the case, then the low penetration of long-term care insurance can certainly be partially explained by the different levels of government in Canada

\(^2\) Transaction costs which represents 33% of the total premium are not extraordinarily high.
offering valued and valuable LTC services, which are crowding-out the private insurance sector. This can occur even if LTC services and coverage are valued.

Brown and Finkelstein (2008) show that social insurance, and in particular Medicaid in the United States, crowds out the demand for private insurance. While acknowledging that the public provision of health services late in life can explain the lack of insurance, Boyer and Glenzer (2017) propose that generous retirement programs (such as the Canadian Pension Plan in Canada, la Régie des rentes du Québec, or Social Security in the United States) also reduce the need for LTC insurance. They contend that high-risk individuals (i.e., those who have a high probability of having a long life) are being subsidized in the retirement and annuity market by low-risk individuals. When time comes for the high-risk individuals to purchase LTC insurance, they realize that they are richer than they should have been had their retirement not been subsidized by the low-risk individuals, and their need for LTC insurance is reduced. Low-risk individuals, seeking to separate themselves from the high-risk individuals may actually be better off not purchasing LTC insurance than subsidizing the high risk individuals a second time. The combination of generous retirement programs run by the government with adverse selection with respect to the risk of living long result in high-risk individuals wanting to be under-insured, and low-risk individuals to have little or even no insurance at all.

2.4 Family (informal help)

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3See next section for a description of the Canadian situation.
For selected countries and regions, and for different dates between 1996 and 2006, marginal impact on the probability that one is employed given 1- that one provides care at home to a family member (Panel A) and 2- the log of the number of hours spent providing care at home to a family member (Panel B)

Source: OECD estimates based on HILDA for Australia, BHPS for the United Kingdom, SHARE for Continental Europe, and HRS for the United States (see OECD 2011).

3 The Model

The originality of the model we propose in this paper comes from the juxtaposition of 1- the market for long-term care services in private facilities, 2- the market for private long-term care insurance, 3- the public provision of long-term care services, and 4- the family provision of informal help. To our knowledge, this is the first time that these four sources of services with respect to long-term care are included in one single model framework.

3.1 The Market for services

The long-term care market is characterized by the juxtaposition of many players who play strategically knowing that all other do the same. In particular, think of the markets for long-term care services and
insurance which are related but not integrated: . We say that the markets are not integrated because it is quite possible for an individual to purchase long-term care services without having purchased LTC insurance. As both markets exist on their own, there must be some supply and demand functions in each market such that an equilibrium is reached that gives us an equilibrium quantity and price for long-term care services and for long-term care protection. Our approach draws some of its inspiration from the health care and health insurance market structure described in Phelps (2010). We will therefore develop equilibrium price and quantities of long-term care services and of long-term care insurance. We depart slightly from the reduced-form approach of Phelps to put more structure to the problem. In that sense our approach is very close to that of Nell et al. (2009) whose results are a special case of the model we propose herein.

Because of the similarities between our model and that of Nell et al. (2009), we will try to stay as close as possible to their notation. The economy we consider is presented in the following diagram. First, agents purchase a long-term care insurance contract from a competitive insurers. This contract reimburses (partially, perhaps) the agent for expenses incurred if he needs long-term care services. After the contract is purchased, Nature chooses with probability $1 - \rho$ a state of the world when the agent does not need long-term care services (outcome 3a). With probability $\rho$, the agent needs some kind of long-term care (outcome 3b). Given the agent needs long-term care services, Nature chooses again the severity of the agent’s need. With probability $\xi_L$, the agent has need $R_L$, with $L \in \{1, 2, \ldots\}$ being some ranking in terms of the amount of services needed (we can think of the number of activities of daily living or instrumental activities of daily living, i.e. ADL and iADL, for which the agent needs some help). Without loss of generality we will let $R_1 < R_2 < R_3 < \ldots$, and for ease of exposition, we will let the probability of needing help with one ADL be given by $\xi_1$, the probability of needing help with two ADLs be given by $\xi_2 = (\xi_1)^2$, and the probability of needing help with all $m$ possible ADLs be given by $\xi_m = (\xi_1)^m$, with $\sum_{L=1}^{m} \xi_L = \sum_{L=1}^{m} \xi^L = 1$. The cost of providing long-term care services in market $L$ is $c_L$, with $c_1 < c_2 < \ldots < c_m$. This cost is the same for all firms providing the service. The cost to the government if some multiple of the cost to the private sector, $\gamma c_L$, with $\gamma > 0$ (and most likely $\gamma \geq 1$). The expected cost of long-term care services is equal to $E(LTC) = \rho \sum_{L=1}^{m} \xi_L c_L$.

The market for long-term care services is cone-shaped, with the provision of government services being located at the top of the cone. The vertical distance between one particular long-term care market and the government is given by $v_L$. The total distance from agent $\mathbf{r}$ to the government’s provision of long-term care services is equal to $G_2 = \sqrt{R_2^2 + v_2^2}$. The government’s social policy is represented by the angle $g \in (0, 180)$ at the top of the cone. The smaller is $g$, the greater is the vertical distance between the government and a

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4Activities of daily living (ADL) typically include the following: walking, bathing, dressing, toileting (including managing incontinence), brushing teeth, eating. Instrumental activities of daily living (iADL) are the activities that people do as members of society such as: managing finances, cooking, driving, communicating (which includes using the telephone, the computer, and iPADS), shopping, managing medication, keeping appointments. Alternatively, the American Occupational Therapy Association (https://www.aota.org/) identifies 12 types of instrumental activities of daily living that may be performed in a community, such as taking care of pets, observing religious rites, or taking care of others.
long-term care market of radius $R_L$. From basic trigonometry, we recall that $\tan \frac{\theta}{2} = \frac{R_L}{v_L}$ and that $\sin \frac{\theta}{2} = \frac{R_L}{G_L}$ so that, for example, the total distance from agent to the government’s provision of long-term care services is equal to $G_2 = \sqrt{R_2^2 + \left(\frac{R_2}{\tan \frac{\theta}{2}}\right)^2} = R_2 \sqrt{1 + \frac{\cos \frac{\theta}{2}^2}{\sin \frac{\theta}{2}^2}} = R_2 \sqrt{\frac{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}}} = R_2 \frac{1}{\sin \left(\frac{\theta}{2}\right)} \in (R_2, \infty)$. In general, the total distance to government services from market $L$ will be $G_L = \frac{R_L}{\sin \frac{\theta}{2}}$. As government services increase (that is, as $g$ becomes larger), then the distance to travel decreases since $\frac{dG_L}{dg} = -\frac{R_L}{2} \frac{\cos \left(\frac{\theta}{2}\right)}{\sin^2 \left(\frac{\theta}{2}\right)} < 0$, and it decreases at a decelerating pace as $g$ becomes greater since $\frac{d^2G_L}{dg^2} = \frac{R_L}{2} \frac{\cos^2 \left(\frac{\theta}{2}\right)}{\sin^3 \left(\frac{\theta}{2}\right)} + \frac{R_L}{4} \frac{1}{\sin \left(\frac{\theta}{2}\right)} > 0$.

![Figure 5: View from the side of the long-term care service diagram of our economy. Severity of the illness (and of the care needed) is given by $R_L$, with $L \in \{1, 2, \ldots\}$. The provision of services by the Government is at the top of the cone so that the distance from one point located on one long-term care service market is given by $G_L = \sqrt{R_1^2 + v_L^2}$, with $R_L$ being the radius of the long-term care market circle and $v_L$ being the vertical distance from the top of the cone to the market. The angle at the top of the cone is given by $g \in (0, 180)$. The total distance is equal to $G_L = R_L \frac{1}{\sin \frac{\theta}{2}}$. This means that the greater is $g$, the smaller is the vertical and total distance.

At stage 1 each agent purchases an insurance contract that maximizes his expected utility. At stage 2 Nature decides whether an agent is sick with probability $\rho$ or not with probability $1 - \rho$. If healthy (stage 3a), consumption occurs and the game ends. If the agent is sick (stage 3b), Nature plays again and chooses both the severity of the care that is needed ($R_L$, with $L \in \{1, 2, \ldots, n\}$ with $\sum_{L=1}^{n} \xi_L = \sum_{L=1}^{n} \xi^L = 1$) and where the ill agent falls on the circle corresponding to his level of illness according to a uniform distribution
over the radian \( U(0, 2\pi) \). Agent must then travel to the closest provider to obtain services \( (S_{L}^{1}, with \ l \in \{1, 2, ...\}) \). The providers of long-term care services in this diagram are represented by the lozenges. Providers compete with others on a given circle, but not across circles because those are different levels of care. When viewed from the top, the provision of services by the Government \( (G) \) appear to be modelled by positioning the government at the center of the circles. Every provider must take into account the presence of the government which provides some level of care.

In this approach, an agent who finds himself at point \( S_{L}^{1} \) would need to seem long-term care services from either provider \( S_{L}^{1} \), provider \( S_{L}^{2} \) or from the government located at point \( G \). Travelling to the government’s location will cost \( R_{L} \) if \( g = 180 \), whereas the circumference of the location of the level-2 providers of long-term services is \( 2\pi R_{L} \). We assume that the unit cost of travel to any private provider of long-term care services to be equal to \( t > 1 \). This means that any one individual should be better off with government services provided that the total cost this individual faces – which includes the travel cost to the government’s location plus the price of the government service, which we assume equal to a factor \( \gamma > 0 \) of the private cost of providing the service \( (c_{L}) \) – is lower than the cost of travelling to the nearest private provider of services plus the price of the private long-term care service \( (p_{L}) \). Mathematically, an agent located at a distance \( d_{L} \)

\[
E(d_{L}) = 2\pi \frac{360}{\pi} \left( \frac{360}{\pi} \right)^{\frac{1}{2}} \left( \frac{1}{n_{L}} \right)^{2} \pi R_{L}.
\]

If we worked in radians instead of degrees, the number of radians that separate any two providers of long-term care services on a given level will be \( \gamma d_{L} = \frac{360}{\pi} \). The average degree to travel will thus be \( E(\gamma d_{L}) = \left( \frac{360}{\pi} \right)^{\frac{1}{2}} \left( \frac{1}{\pi} \right) \), which translates into

\[
E(d_{L}) = \frac{2\pi}{360} \left( \frac{360}{\pi} \right)^{\frac{1}{2}} \left( \frac{1}{n_{L}} \right)^{2} \pi R_{L}.
\]

In terms of actual distance travelled along the circumference, it will be equal to

\[
E(d_{L}) = E(\gamma d_{L}) R_{L} = \left( \frac{2\pi}{n_{L}} \right)^{\frac{1}{2}} \left( \frac{1}{n_{L}} \right) R_{L} = \left( \frac{1}{n_{L}} \right)^{2} \pi R_{L}.
\]

Obviously, both are equal.

### 3.2 Services from the government or from the private sector?

Without loss of generality, let us set the cost per unit of distance travelled to the government’s point of service to 1. This means that the total travel cost to government services from market of level \( L \) is

\[
G_{L} = R_{L} \frac{1}{2} \neq R_{L} \sqrt{\frac{2}{1 - \cos g}}. \quad \text{We assume that the unit cost of travel to any private provider of long-term care services to be equal to } t > 1. \text{ This means that any one individual should be better off with government services provided that the total cost this individual faces – which includes the travel cost to the government’s location plus the price of the government service, which we assume equal to a factor } \gamma > 0 \text{ of the private cost of providing the service } (c_{L}) – \text{ is lower than the cost of travelling to the nearest private provider of services plus the price of the private long-term care service } (p_{L}). \text{ Mathematically, an agent located at a distance } d_{L}.
\]

\[\footnote{We could, possibly, presume that there is a one-to-one correspondence between } g \text{ and } \gamma \text{ so that the greater the cost of government services } (\gamma), \text{ the further away these services appears to be } (g). \text{ Let us get away from that problem and presume that } \gamma \text{ is very close to } 1, \text{ so that } \frac{1}{\sin^{2} R_{L}} >> (1 - \gamma) c_{L}. \]
Figure 6: View from above of the diagram of our economy. At stage 1 each agent purchases an insurance contract that maximizes his expected utility. At stage 2 Nature decides with probability $\rho$ whether an agent is sick or not. If healthy (stage 3a), consumption occurs. If the agent is sick (stage 3b), Nature plays again and chooses the severity of the care that is needed ($R_L$, with $L \in \{1, 2, \ldots\}$) and where the ill-agent falls on the circle corresponding to his level of illness. Agent must then travel to the closest provider to obtain services ($S^L_i$, with $l \in \{i, j, \ldots\}$). Viewed from above, the provision of services by the Government appear to be positioned at the center of the circles.
of the nearest private provider of long-term care services will strictly prefer to consume his long-term care services from a private provider if and only if \( td_L + p_L < \frac{R_L}{t(\sin \frac{\pi}{2})} + \gamma c_L \).

**Proposition 1** In a market without insurance, private provision of long-term care services paid out-of-pocket will be strictly preferred if the distance an agent must travel to reach the location of a private provider is given by \( d_L < \frac{R_L}{t(\sin \frac{\pi}{2})} - \frac{p_L - \gamma c_L}{t} \).

*Proof: Straightforward.*

We know that the expected distance to travel is \( E(d_L) = \pi \left( \frac{1}{n_L} \right)^2 R_L \), which is also the median distance to travel because an agent’s location in the market is drawn from a uniform distribution over the radian \( U(0, 2\pi) \). Because of the uniform distribution, we know the maximum distance that any agent will travel for a private provision of long-term care services will be \( \text{Max}(d_L) = 2\pi \left( \frac{1}{n_L} \right)^2 R_L \). Let \( z \) represent the fraction of the population with LTC needs \( R_L \) that is better of receiving LTC services from the government than the private sector. This means that in this economy, in this market, the government will provide services to no individual in need of long-term care services (that is, \( z = 0 \)) if \( 2\pi \left( \frac{1}{n_L} \right)^2 R_L < \frac{R_L}{t(\sin \frac{\pi}{2})} - \frac{p_L - \gamma c_L}{t} \), which will occur if the price of private long-term care services is such that \( p_L < \gamma c_L + \left( \frac{1}{\sin \frac{\pi}{2}} - \frac{3}{2} \pi \left( \frac{1}{n_L} \right)^2 \right) R_L \).

This allows us to make three point estimates regarding the relative importance of the public sector in the provision of long-term care services.

- In order for less than one-quarter of the population to receive treatment from the government, we need to have \( \frac{3}{2} \pi \left( \frac{1}{n_L} \right)^2 R_L < \frac{R_L}{t(\sin \frac{\pi}{2})} - \frac{p_L - \gamma c_L}{t} \), which means that the price of long-term care services in the private sector must be such that \( p_L < \gamma c_L + \left( \frac{1}{\sin \frac{\pi}{2}} - \frac{3}{2} \pi \left( \frac{1}{n_L} \right)^2 \right) R_L \).

- In order for less than half of the population to receive treatment from the government, we need to have \( \frac{1}{2} \pi \left( \frac{1}{n_L} \right)^2 R_L < \frac{R_L}{t(\sin \frac{\pi}{2})} - \frac{p_L - \gamma c_L}{t} \), which means that the price must be such that \( p_L < \gamma c_L + \left( \frac{1}{\sin \frac{\pi}{2}} - \frac{1}{2} \pi \left( \frac{1}{n_L} \right)^2 \right) R_L \).

- In order for less than three-quarter of the population to receive treatment from the government, we need to have \( \frac{1}{2} \pi \left( \frac{1}{n_L} \right)^2 R_L < \frac{R_L}{t(\sin \frac{\pi}{2})} - \frac{p_L - \gamma c_L}{t} \), which means that the price must be such that \( p_L < \gamma c_L + \left( \frac{1}{\sin \frac{\pi}{2}} - \frac{1}{2} \pi \left( \frac{1}{n_L} \right)^2 \right) R_L \).

**Proposition 2** In general, for the proportion of the population that receives treatment from the government to be less than \( z \) (and a proportion \( 1 - z \) of the population receives its LTC services from the private sector), we need to have \( 2(1 - z) \pi \left( \frac{1}{n_L} \right)^2 R_L < \frac{R_L}{t(\sin \frac{\pi}{2})} - \frac{p_L - \gamma c_L}{t} \). The market price of privately provided long-term care

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\(^6\)Note that if the unit cost to travel to the point where the government is providing services was given by \( t_g \neq 1 \), then the private provision of long-term care services would be preferred provided that \( t < t_g \frac{R_L}{t(\sin \frac{\pi}{2})} - \frac{p_L - \gamma c_L}{t} \).
services must then be such that \( p_L < \gamma c_L + \left( \frac{1}{\sin \frac{\pi}{2}} - 2\pi (1 - z) \right) \left( \frac{1}{n_L} \right)^2 R_L \). Equivalently, the proportion of individuals who, absent any long-term care insurance market, will prefer to obtain their long-term care services from a government entity is given by

\[
z = 1 - \frac{(n_L)^2}{2\pi(R_L)^2} - \frac{\gamma c_L(n_L)^2}{2\pi R_L} + \frac{(n_L)^2}{2\pi R_L} p_L ,
\]

which is linear and increasing in the price of privately provided long-term care services.

3.3 Equilibrium price, number service providers, and the provision of government services

We know from the solution to a Salop circular city model of dimension \( \Gamma_L = 2\pi R_L \) that, provided there are \( n \) suppliers in a given market, the equilibrium price charged by each equidistant supplier will be \( p_L = c_L + \frac{\Gamma_L t}{n_L} \).

We can also find that, given open entry at cost \( f \), the number of firms that will enter the market such that no profit is made in equilibrium will be equal to \( n_L^* = \Gamma_L \sqrt{\frac{z}{2}} = 2\pi R_L \sqrt{\frac{z}{2}} \). We thus have that the equilibrium number of firms \( n_L^* \) grows linearly with the size of the market \( (\Gamma_L = 2\pi R_L) \).

**Proposition 3** The equilibrium price charged by each equidistant supplier will be \( p_L^* = c_L + \frac{\Gamma_L t}{n_L} = c_L + \sqrt{tf} \), and is independent of the size of the market, \( R_L \).

**Proof:** Straightforward.

If there is no government in the economy (the equivalent of having \( g = 0 \)), then the expected out-of-pocket expense for long-term care services will be \( E(p^*) = \rho \sum_{L=1}^{n} \xi_L p_L = \rho \sum_{L=1}^{n} \xi_L (c_L + \sqrt{tf}) \).

No agent in the population of a given market will therefore be serviced by the government provided that

\[
p_L^* \leq \gamma c_L + \left( \frac{1}{\sin \frac{\pi}{2}} - 2\pi \left( \frac{1}{n_L} \right)^2 \right) R_L .
\]

Substituting for \( n_L^* = 2\pi R_L \sqrt{\frac{z}{2}} \) and \( p_L^* = c_L + \sqrt{tf} \) gives us that no agent will be serviced by the government if and only if

\[
(1 - \gamma) c_L + \sqrt{tf} \leq \frac{1}{\sin \frac{\pi}{2}} R_L - \frac{f}{2\pi R_L}.
\]

This occurs when \( 2\pi (R_L)^2 - 2\pi \left( (1 - \gamma) c_L + \sqrt{tf} \right) \left( \sin \frac{\pi}{2} \right) R_L - f \sin \frac{\pi}{2} \geq 0 \). Note that if entry is free \( (f \to 0) \), then no one will receive services from the government provided that \( R_L \geq (1 - \gamma) c_L \sin \frac{\pi}{2} \).

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7 Recall from the Salop model that each firm on market \( R_L \) of size \( \Gamma_L = 2\pi R_L \) maximizes problem

\[
\max_{p_L} \Pi_L = (p_L - c_L) \left( \frac{\Gamma_L}{n_L} + \frac{p_L - p_L}{t} \right) - f
\]

which had first order condition

\[
\frac{\partial \Pi_L}{\partial p_L} = \left( \frac{\Gamma_L}{n_L} + \frac{p_L - c_L}{t} \right) - \left( \frac{p_L - c_L}{t} \right) = 0
\]

so that

\[
\frac{\Gamma_L t}{n_L} + c_L = 2 p_L - p
\]

With all firms symmetric, we have \( p_L = \overline{p} \) such that \( p_L = c_L + \frac{\Gamma_L t}{n_L} \).

8 So that \( \Pi_L (p_L^*) = (p_L^* - c_L) \left( \frac{\Gamma_L}{n_L} + \frac{p_L^* - p_L^*}{t} \right) - f = 0 \), which, knowing that \( p_L^* = \overline{p} \) by symmetry and \( p_L^* = c_L + \frac{\Gamma_L t}{n_L} \) gives us \( \Pi_L (p_L^*) = \frac{\Gamma_L}{n_L} \left( \overline{p} \right) - f = 0 \) and \( n_L^* = \Gamma_L \sqrt{\frac{z}{2}} \).
it is as costly for the government to offer LTC services as for the private sector (so that \( \gamma = 1 \)), then all LTC services will be provided by the private sector. A fortiori, if it is more costly for the government to provide the service than for the private sector (so that \( \gamma > 1 \)), then for sure free entry removes the need for any government provided long-term care services.

Finding the zeros of this function for \( f > 0 \) give us that private provision of LTC services is the norm for all when\(^9\)

\[
R_L > \frac{1}{2} \left((1-\gamma) c_L + \sqrt{tf} \right) \sin \frac{g}{2} + \sqrt{\frac{1}{4} \left( \left( (1-\gamma) c_L + \sqrt{tf} \right) \sin \frac{g}{2} \right)^2 + \frac{f}{2\pi} \sin \frac{g}{2}}.
\]

Recalling that \( \sin \frac{g}{2} \in (0,1) \), it is then clear that, ceteris paribus, no agent in a particular market will be serviced by the government if the severity of the needs is important (that is, \( R_L \) is large when there are more ADLs), when government services are further (that is, when \( g \) is small), when the cost of travelling is low (that is, when \( t \) is small), when entry cost is low (that is, when \( f \) is small) such that there are numerous firms in the economy (that is, when \( n_L^* \) is large), and when the difference between the cost to the private sector and the cost to the government of offering services is small (that is, when \( (1-\gamma) c_L \) is small).

Interestingly, the cost of providing services, \( c_L \), is a part of this decision only because, by assumption, the cost to the government is not the same as for the private sector (i.e., \( \gamma \) is not necessarily equal to 1). The impact of the cost of providing the service is therefore not known by itself; we need to know whether \( (1-\gamma) \) is positive or negative. If it is positive (resp. negative), which would mean that the government faces lower (resp. higher) cost of offering the same LTC service as the private sector, then the greater is the cost of providing services, the more likely will such services be offered by the government (resp. private sector). Similar to what we presented in the previous section, we can make three point estimates regarding the relative importance of the public sector in the provision of long-term care services based, here, on the severity of the long-term care needs.

- In order for less than one-quarter of the population to receive treatment from the government, we need to have \( c_L + \sqrt{tf} \leq \gamma c_L + \left( \frac{1}{\sin \frac{g}{2}} - \left( \frac{3}{4} \right) \left( 2\pi \left( \frac{1}{n_L} \right)^2 \right) t \right) R_L \). This occurs when

\[
(1-\gamma) c_L + \sqrt{tf} \leq \frac{1}{\sin \frac{g}{2}} R_L - \left( \frac{3}{4} \right) \left( \frac{f}{2\pi R_L} \right)
\]

\[
2\pi (R_L)^2 - ((1-\gamma) c_L + \sqrt{tf}) \sin \frac{g}{2} R_L - \left( \frac{3}{4} \right) f \sin \frac{g}{2} \geq 0
\]

\[
R_L > \frac{1}{2} \left((1-\gamma) c_L + \sqrt{tf} \right) \sin \frac{g}{2} + \sqrt{\frac{1}{4} \left( \left( (1-\gamma) c_L + \sqrt{tf} \right) \sin \frac{g}{2} \right)^2 + \left( \frac{3}{4} \right) \frac{f}{2\pi} \sin \frac{g}{2}}.
\]

\[^9\]The second possibility is to have

\[
R_L < \frac{1}{2} \left((1-\gamma) c_L + \sqrt{tf} \right) \sin \frac{g}{2} - \sqrt{\frac{1}{4} \left( \left( (1-\gamma) c_L + \sqrt{tf} \right) \sin \frac{g}{2} \right)^2 + \left( \frac{3}{4} \right) \frac{f}{2\pi} \sin \frac{g}{2}}
\]

which would have \( R_L < 0 \), an impossibility.
In order for less than half of the population to receive treatment from the government, we need to have
\[ c_L + \sqrt{t_f} < \gamma c_L + \left( \frac{1}{\sin \frac{g}{2}} - \frac{1}{2} \left( 2\pi \left( \frac{1}{n_L} \right)^2 \right) t \right) R_L. \]
This occurs when
\[ R_L > \frac{1}{2} \left( (1 - \gamma) c_L + \sqrt{t_f} \right) \sin \frac{g}{2} + \sqrt{\frac{1}{4} \left( \left( (1 - \gamma) c_L + \sqrt{t_f} \right) \sin \frac{g}{2} \right)^2 + \left( \frac{1}{2} \right) \frac{f}{2\pi} \sin \frac{g}{2}. \]

In order for less than three-quarters of the population to receive treatment from the government, we need to have \( c_L + \sqrt{t_f} \leq \gamma c_L + \left( \frac{1}{\sin \frac{g}{2}} - \frac{1}{4} \left( 2\pi \left( \frac{1}{n_L} \right)^2 \right) t \right) R_L. \) This occurs when
\[ R_L > \frac{1}{2} \left( (1 - \gamma) c_L + \sqrt{t_f} \right) \sin \frac{g}{2} + \sqrt{\frac{1}{4} \left( \left( (1 - \gamma) c_L + \sqrt{t_f} \right) \sin \frac{g}{2} \right)^2 + \left( 1 - z \right) \frac{f}{2\pi} \sin \frac{g}{2}. \]

The same general comparative statics as before hold.

We can now turn the equation around and say that, given long-term care needs \( R_L, \) cost of entry in the market for long-term care services \( f, \) cost of travelling to long-term care facilities \( t, \) cost of private services \( c_L, \) government involvement \( g, \) and cost factor of providing long-term care services \( \gamma, \) the proportion of the population which will receive government services, when no long-term care market exists, is given by \( z^\theta \) such that
\[ z^\theta = 1 + 2\pi \frac{R_L}{f} \left( \sqrt{t_f} + (1 - \gamma) c_L - \frac{1}{\sin \frac{g}{2}} R_L \right). \]

It is clear that this proportion of services obtained from the government is not linear in all parameters.

In particular, we note the quadratic factor in long-term care needs \( (R_L) \) and the square-root impact of the travel cost to obtain private services \( (\sqrt{t_f}). \) In the next proposition, we examine the impact of each parameter on the proportion of individuals that receive public long-term care services.

**Proposition 4** In a market characterized by competitive suppliers of long-term care services and public provision of those same services, and absent any insurance to cover the cost of private long-term care services, individuals receiving services from the public sector will be larger when:

1. the smaller is the need for services, \( R_L, \) if and only if \( R_L > \frac{1}{2} \left( (1 - \gamma) c_L + \sqrt{t_f} \right) \sin \frac{g}{2}, \)\(^{10}\) with \( \frac{\partial z^\theta}{\partial R_L} < 0. \)
2. the greater is the cost of finding a private provider, \( t, \) with \( \frac{\partial z^\theta}{\partial t} < 0; \)

\(^{10}\) Or, equivalently, if the number of suppliers in equilibrium is such that \( n_L^* = \sqrt{\frac{2\pi R_L t}{f}} > \frac{\pi}{2} \sqrt{\frac{1 - \cos g}{g}}. \)
3- the smaller (resp. greater) is the marginal cost of services, $c_L$ if $\gamma < 1$ (resp. $\gamma > 1$), with $\frac{\partial^2 z}{\partial c_L^2} = 0$;  
4- the smaller is the cost premium for government services, $\gamma$, with $\frac{\partial^2 z}{\partial \gamma^2} = 0$;  
5- the closer is the government or the greater is its investment in social services, $1^{11} g$, with $\frac{\partial^2 z}{\partial g^2} < 0$; and  
6- the greater is the cost of entry $f$, if and only if $R_L > \left( (1 - \gamma) c_L + \frac{1}{2} \sqrt{f/t} \right) \sin \frac{g}{2}$.

Proof: Straightforward since for any parameter $x \in \{R_L, f, t, c_L, g, \gamma\}$, we only need to find the partial derivative $\frac{\partial z}{\partial x}$.

4 Market for insurance

In the previous section, we assumed that individuals could not buy insurance against the risk of having to pay for long-term care services. We will not introduce the possibility of having insurance, first through a coinsurance clause such that the individual pays proportion $\delta$ of the final cost, through a deductible clause such that the individual pays the first $D$ dollars of the treatment, and then through a "lottery-winning" scheme whereby individuals receive a lump-sum payment independent of the distance they need to travel to the closest provider of long-term care services ($t$), but dependent on the severity of their ADL needs ($R_L$).

4.1 LTC insurance with co-insurance

In the case of the coinsurance contract, the agent ends up paying out-of-pocket a price $\delta p_L$ for services of quality $R_L$. Going back to the solution to a Salop circular city model of dimension $L = 2\pi R_L$, the equilibrium price charged by each equidistant supplier will then be $p_L^\# = c_L + \frac{\Gamma_L}{\delta n_L}$. Given open entry cost of $f$, the equilibrium number of firms will be equal to $n_L^\# = \frac{\Gamma_L}{\pi f} > n_L^*$.  

Proposition 5 The equilibrium price charged by each equidistant supplier will be $p_L^\# = c_L + \sqrt{\frac{f}{\delta}} > p_L^*$.  

Proof: Each firm on market $R_L$ of size $\Gamma_L$ maximizes problem

$$\max_{p_L} \Pi_L = (p_L - c_L) \left( \frac{\Gamma_L}{n_L^\#} + \frac{\delta (p - p_L)}{t} \right) - f$$

$$\frac{\partial z}{\partial \gamma} = \frac{2\pi}{f} (R_L)^2 \frac{\cos \frac{g}{2}}{(\sin \frac{g}{2})^2} > 0$$

$^{12}$Here, each firm on market $R_L$ of size $\Gamma_L$ maximizes problem

$$\max_{p_L} \Pi_L = (p_L - c) \left( \frac{\Gamma_L}{n_L} + \frac{\delta (p - p_L)}{t} \right) - f$$

which has a first order condition such that

$$\frac{\Gamma_L t}{\delta n} + c = 2p_L - \bar{p}$$

With all firms symmetric, we have $p_L = c + \frac{\Gamma_L t}{\delta n}$.
since the policyholder pays only fraction $\delta$ of the total price. The first order condition of this problem is such that

$$\frac{\Gamma_L t}{\delta n_L} + c_L = 2p^#_L - \bar{p}$$

With all firms symmetric (so that $p^#_L = p$ for all insurers), we have $p^#_L = c_L + \frac{\Gamma_L t}{\delta n_L}$. Clearly this is greater than the price without insurance, $p^*_L = c_L + \frac{\Gamma_L t}{\delta n_L}$, when $\delta < 1$. □

Assume no government in the economy (the equivalent of having $g = 0$). If agents have the possibility to acquire insurance against the cost of long-term care services, then the expected out-of-pocket expense for the agent of acquiring long-term care services, assuming coinsurance rate of $\delta$ and an anticipated price for private long-term care services of $p^*_L$, will be $E(\delta p^#) = \rho \sum_{L=1}^{m} \xi_L \delta \left( c_L + \sqrt{\frac{L}{\delta}} \right)$. The expected insurance premium for this policy, assuming no loading other than the ones endogenously determined, is thus $\pi^# = \rho \sum_{L=1}^{m} \xi_L \left( 1 - \delta \right) \left( c_L + \sqrt{\frac{L}{\delta}} \right)$.

Knowing that $n^#_L = \Gamma_L \sqrt{\frac{1}{4\pi}}$ and that $\Gamma_L = 2\pi R_L$ for given level of ADL needs, we infer that no agent in the population will be serviced by the government if the travel cost to the nearest private provider of long-term care services, $td_L$, plus the co-insured price of these services, $\delta p^#_L = \delta \left( c_L + \frac{\Gamma_L t}{\delta n_L} \right) = \delta \left( c_L + \sqrt{\frac{2\pi R_L}{\delta}} \right)$, is smaller than the uninsured cost of government services, $R_L + \gamma c_L$. We recall that the average and median travel distance are given by $E(d_L) = \pi \left( \frac{1}{n^#_L} \right)^2 R_L$, so that the expected and median travel cost is given by $E(td_L) = \pi t \left( \frac{1}{2\pi R_L \sqrt{\pi}} \right)^2 R_L = \frac{\delta f}{4\pi R_L}$ with a maximum travel cost of $Max(td_L) = 2E(td_L) = \frac{\delta f}{2\pi R_L}$. Again, both the average travel cost and the maximum travel cost are independent of the unit cost of travel, $t$. We therefore have that no agent will be serviced by the government if even the agent that is the furthest from a private provider prefers to purchase his services from the private provider than from the government. In other words, no agent receives government services provided that

$$\delta p^#_L + Max(td_L) < \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L$$

$$\delta c_L + \sqrt{\delta t f} + \frac{\delta f}{2\pi R_L} < \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L$$

$$\left( c_L + \frac{f}{2\pi R_L} \right) \left( \sqrt{\delta} \right)^2 + \left( \sqrt{tf} \right) \left( \sqrt{\delta} \right) - \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right) < 0$$
Noting that this is a parabola in with respect to $\sqrt{\delta}$, we have that private provision of insured long-term care services are always preferred by all agents if and only if\(^\text{13}\)

$$\sqrt{\delta} < \frac{-\sqrt{tf} + \sqrt{tf + 4 \left(c_L + \frac{f}{2\pi R_L} \right) \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right)}}{2 \left(c_L + \frac{f}{2\pi R_L} \right)}$$

We note that this is certainly true in the case of full insurance, which is associated with $\delta = 0$, since the fraction on the right hand side of this inequality is clearly positive.

We can now wonder what is the impact of insurance coverage on the proportion of individuals who seek services from the state. As in the case without insurance (where $\delta = 1$) let $z^#$ be the proportion of the population that receives treatment from the government (and $1 - z^#$ be the proportion that receives long-term care services from the private sector). Recalling that $p^#_L = c_L + \frac{R_L}{\delta n^#_L}$, that $n^#_L = \Gamma_L \sqrt{\frac{f}{\delta}}$, and that $\Gamma_L = 2\pi R_L$, we have that the cutoff point $z^#$ is given by

$$\delta p^#_L + (1 - z^#) \left( \frac{1}{n^#_L} \right)^2 \Gamma_L t = \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L.$$ 

Substituting for $p^#_L$, $n^#_L$, and $\Gamma_L$, and simplifying, we find

$$\delta c_L + \sqrt{\delta ft} + (1 - z^#) \left( \frac{\delta f}{2\pi R_L} \right) = \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L$$

and thus

$$z^# = 1 + \frac{2\pi}{f} R_L \left( \sqrt{\frac{ft}{\delta}} + \left(1 - \frac{\gamma}{\delta} \right) c_L - \frac{1}{\delta} \left( \frac{1}{\sin \frac{\delta}{2}} \right) R_L \right)$$

which compares to the situation without insurance where

$$z^0 = 1 + \frac{2\pi}{f} R_L \left( \sqrt{ft} + \left(1 - \frac{\gamma}{\delta} \right) c_L - \frac{1}{\sin \frac{\delta}{2}} R_L \right)$$

**Proposition 6** Assuming a coinsurance provision of $\delta \leq 1$, then the proportion of individuals that will choose to purchase their long-term care services from the government is given by

$$z^# = 1 + \frac{2\pi}{f} R_L \left( \sqrt{\frac{ft}{\delta}} + \left(1 - \frac{\gamma}{\delta} \right) c_L - \frac{1}{\delta} \left( \frac{1}{\sin \frac{\delta}{2}} \right) R_L \right).$$

The market price of privately provided long-term care services is given by $p^#_L = c_L + \sqrt{\frac{f}{\delta}}$ and the number of insurers is equal to $n^#_L = 2\pi R_L \sqrt{\frac{f}{\delta}}$, both of which are greater than when there is no insurance (i.e., when $\delta = 1$). The amount paid by the agent, $\delta p^#_L$ is, however, smaller when there is insurance since $\delta p^#_L = \delta c_L + \sqrt{\delta ft} < c_L + \sqrt{ft} = p^#_L$ for any $\delta < 1$.

\(^{13}\)In the $\delta$-space, this inequality is equivalent to

$$\delta < \frac{-\sqrt{tf} + \sqrt{tf + 4 \left(c_L + \frac{f}{2\pi R_L} \right) \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right)}}{2 \left(c_L + \frac{f}{2\pi R_L} \right)} - \frac{\sqrt{(tf)^2 + 4ft \left(c_L + \frac{f}{2\pi R_L} \right) \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right)}}{4 \left(c_L + \frac{f}{2\pi R_L} \right)^2}.$$
Is the proportion of individuals that rely on government services greater or smaller when there is private long-term care insurance? To answer that question, let us look at how $z^\#$ varies when $\delta$ varies. Taking the partial derivative of $z^\#$ with respect to $\delta$ gives us

$$\frac{\partial z^\#}{\partial \delta} = \frac{2\pi}{f^2} R_L \left( -\frac{1}{2} \sqrt{\delta f t} + \gamma c_L + \left( \frac{1}{\sin \frac{\delta}{2}} \right) R_L \right)$$

This derivative is positive, meaning that government services are more popular ($z^\#$ goes up) when insurance coverage decreases ($\delta$ goes up) when

$$-\frac{1}{2} \sqrt{\delta f t} + \gamma c_L + \left( \frac{1}{\sin \frac{\delta}{2}} \right) R_L > 0$$

This is more likely to occur when $\delta$, $f$, $t$, and $g$ (respectively, when out-of-pocket payments, the cost of entry, the travel cost, and the distance to the government) are small, and when $\gamma$, $c_L$, and $R_L$ (respectively, when the cost premium for government services, the cost of long-term care services, and the extent of long-term care needs) are large.

4.2 LTC insurance with a deductible

In the case of the deductible contract, the agent ends up paying an amount $D$ for services of any quality. The equilibrium is then based on the consumer’s choice between the two providers of long-term care services: $(p_L - D) + td = (p_L - D) + t \left( \frac{1}{n} - d \right)$. Solving this problem gives us a price charged by the private provider of long-term care services which is the same as when $D = 0$. This means that the presence of a deductible does not change any of the analysis of the private long-term care market we have presented thus far. In fact, the market solution is similar to the case where coinsurance is zero (that is, $\delta = 1$). There will, of course, be a difference when it comes to the choice between public and private provision of long-term care services.

We can now wonder what is the impact of insurance coverage on the proportion of individuals who seek services from the state. As in the case without insurance (where $\delta = 1$) let $z^D$ be the proportion of the population that receives treatment from the government under a deductible contract (and $1 - z^D$ be the proportion that receives long-term care services from the private sector). Because the long-term care market equilibrium solution is the same as when coinsurance is zero, we have a price of long-term care services on market $R_L$ equal to $p_L^D = c_L + \frac{\Gamma_L}{n_L^D}$ and a number of providers of services equal to $n_L^D = \Gamma_L \sqrt{\frac{f}{t}}$. Cutoff point $z^D$ is then given by

$$p_L^D + D + \left( 1 - z^D \right) \left( \frac{1}{n_L^D} \right)^2 \Gamma_L t = \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L.$$ 

Substituting for $p_L^D$, $n_L^D$, and $\Gamma_L$, and simplifying, we find

$$c_L + \sqrt{ft} + D + (1 - z^D) \left( \frac{f}{2\pi R_L} \right) = \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L.$$
Isolating $z^D$, we find that the proportion of individuals seeking services from the government is

$$z^D = 1 + \left[ (1 - \gamma) c_L + \sqrt{F_1} - \frac{R_L}{\sin \frac{g}{2}} \right] \left( \frac{2\pi R_L}{f} \right) + D \left( \frac{2\pi R_L}{f} \right).$$

Compared to the case of no deductible ($D = 0$) and no coinsurance ($\delta = 1$), we note that $z^D = z + D \left( \frac{2\pi R_L}{f} \right)$. Consequently, individuals are more likely to purchase their services from the government if they need to pay a deductible in addition to the market price of private long-term care services and the travel cost to these services.

### 4.3 Lump-sum LTC insurance

A third type of long-term care insurance contract that exists in the economy are contracts where a fixed amount is paid to the individual independent of the cost of the services he receives, but dependent on the level of care he needs. In other words, the individual receives an amount $\Delta_L$, which depends on the extent of long-term care services he needs (so that $\Delta_1 < \Delta_2 < \ldots < \Delta_n$). In the setup of our model, this would have no impact whatsoever on the price of private long-term care services nor on the proportion of individuals who seek treatment from the government. The reason is that the lump-sum payment is made based on the individual’s condition, and not on where long-term care services are consumed. In other words, $\Delta_L$ is pocketed by the individual irrespective of who provides the long-term care services. It follows that such an insurance contract would have no impact on the public versus private provision of long-term care services, on the number of private providers, or on the price that these providers charge.

### 5 Optimal insurance coverage

Knowing that the level of coinsurance value is relevant and not trivial, we now examine what would be its optimal level that maximizes the individual’s expected utility. We will then examine the case of the deductible and of the lump-sum award. From a modelling perspective, these last two are much less complicated since we can see them as lottery wins in certain states of the world.

#### 5.1 Choosing the level of co-insurance

In a classic setting where insurers charge the actuarial fair price for the policy, we know that policyholders would like to choose a contract that offers them full insurance. In the current setting, however, where the number of service providers and the price for services are determined endogenously, insurance is not offered at the actuarially fair price. This tells us that full insurance is not likely the optimal solution.

We first examine the problem when the government is not a factor (or equivalently that $g = 0$). Using $W$ as the individual’s endowed wealth, the problem he faces, assuming vonNeumann-Morgenstern expected...
utility over final wealth, with $U'(\bullet) > 0$ and $U''(\bullet) < 0$, is to choose a coinsurance level $\delta$ such that:

$$\max_{\delta} EU(\bullet) = (1 - \rho) U(W - \pi^#) + \rho \sum_{L=1}^{m} \xi_L U\left(W - \pi^# - \delta \left(c_L + \sqrt{\frac{tf}{\delta}}\right)\right)$$

s.t. $\pi^# = \rho \sum_{L=1}^{m} \xi_L (1 - \delta) \left(c_L + \sqrt{\frac{tf}{\delta}}\right)$

Even before examining the first order condition, it is clear that full insurance (i.e., $\delta = 0$) is not optimal here since it would mean that the premium, $\pi^#$, would be infinite. The reason is that, once the individuals do not care anymore about the price of long-term care services (essentially because they no longer bear any of it), providers of these services are able to charge whatever price they want without affecting the demand for their services. This is an issue which has the flavour of moral hazard, with the difference that policyholders do not consume greater quantities of services since, conditional on the level of services needed, it is always equal to one unit. The total expense for the services increases, however, because there are too many providers of this service that are coming into the market and paying the fixed cost of setting up these services. As we recall, the number of long-term care providers is given by $n^# = \sqrt{\frac{t}{\delta f}}$, each of which pays a fixed cost $f$ to enter the market. The total investment by providers of long-term care services is thus $n^# f = \sqrt{\frac{tf}{\delta}}$. It is this investment that skyrockets when $\delta \rightarrow 0$, that is when policyholder are almost fully insured.

### 5.1.1 If full insurance is not a solution; what is?

Anticipating the "moral hazard" impact on the price of long-term care services, insurers increase the premium so that they do not lose money. In the limit, as policyholders are almost fully insured, we have that the insurance premium goes to infinity: $\lim_{\delta \rightarrow 0} \pi^# = \infty$. Note in particular how the equilibrium premium behaves as a function of the level of insurance

$$\frac{\partial \pi^#}{\partial \delta} = -\rho \sum_{L=1}^{m} \xi_L \left(c_L + \left(\frac{1 + \delta}{2\delta}\right) \sqrt{\frac{tf}{\delta}}\right)$$

Keeping in mind that full insurance will not be a solution, the first order condition of the policyholder’s problem under the constraint that the insurer makes no profit in expectation solves

$$\max_{\delta} EU(\bullet) = (1 - \rho) U\left(W - \rho \sum_{L=1}^{m} \xi_L (1 - \delta) \left(c_L + \sqrt{\frac{tf}{\delta}}\right)\right)$$

$$+ \rho \sum_{L=1}^{m} \xi_L U\left(W - \rho \sum_{L=1}^{m} \xi_L (1 - \delta) \left(c_L + \sqrt{\frac{tf}{\delta}}\right) - \delta \left(c_L + \sqrt{\frac{tf}{\delta}}\right)\right)$$

\[14\text{Note that we shall continue using the superscript } \# \text{ to refer to the coinsurance situation.} \]
so that

\[
0 = - (1 - \rho) U' (W - \pi^\#) \left( \frac{\partial \pi^\#}{\partial \delta} \right)
- \rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^\# - \delta \left( c_L + \frac{t_f}{\delta} \right) \right) \left( \frac{\partial \pi^\#}{\partial \delta} + \left( c_L + \frac{1}{2} \frac{t_f}{\delta} \right) \right)
\]

if and only if

\[
0 = - \left[ (1 - \rho) U' (W - \pi^\#) + \rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^\# - \delta \left( c_L + \frac{t_f}{\delta} \right) \right) \right] \frac{\partial \pi^\#}{\partial \delta}
- \rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^\# - \delta \left( c_L + \frac{t_f}{\delta} \right) \right) \left( c_L + \frac{1}{2} \frac{t_f}{\delta} \right)
\]

and

\[
\frac{\partial \pi^\#}{\partial \delta} = - \frac{\rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^\# - \delta \left( c_L + \frac{t_f}{\delta} \right) \right) \left( c_L + \frac{1}{2} \frac{t_f}{\delta} \right)}{(1 - \rho) U' (W - \pi^\#) + \rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^\# - \delta \left( c_L + \frac{t_f}{\delta} \right) \right)}
\]

Letting \( \delta^* \) represent the optimal solution, the coinsurance level of long-term care insurance that maximizes a policyholder’s expected utility is the solution to the following implicit function

\[
\sum_{L=1}^{m} \xi_L c_L + \left( 1 + \frac{\delta^*}{2 \delta^*} \right) \sqrt{\frac{t_f}{\delta^*}} = \frac{\sum_{L=1}^{m} \xi_L U' \left( W - \pi^* - \delta^* \left( c_L + \sqrt{\frac{t_f}{\delta^*}} \right) \right) \left( c_L + \frac{1}{2} \sqrt{\frac{t_f}{\delta^*}} \right)}{(1 - \rho) U' (W - \pi^*) + \rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^* - \delta^* \left( c_L + \sqrt{\frac{t_f}{\delta^*}} \right) \right)}
\]

with \( \pi^* = \rho \sum_{L=1}^{m} \left[ \xi_L \left( c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{t_f}{\delta^*}} \right) \right] \).

If there is only one state of the world when sick, this equality becomes\(^{15}\)

\[
\frac{c_L + \left( \frac{1 + \delta^*}{\sqrt{\delta^*}} \right) \sqrt{\frac{t_f}{\delta^*}}}{c_L + \frac{1}{2} \sqrt{\frac{t_f}{\delta^*}}} = \frac{U' \left( W - \rho \left( c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{t_f}{\delta^*}} \right) \right) - \delta \left( c_L + \sqrt{\frac{t_f}{\delta^*}} \right)}{(1 - \rho) U' \left( W - \rho \left( c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{t_f}{\delta^*}} \right) \right) + \rho U' \left( W - \rho \left( c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{t_f}{\delta^*}} \right) \right) - \delta \left( c_L + \sqrt{\frac{t_f}{\delta^*}} \right)}
\]

which is proof that full insurance is not possible. To see why, note that for full insurance to occur, we would need both sides of this equality to equal one. This is not the case since, on the left, we have \( c_L + \frac{1 + \delta^*}{\sqrt{\delta^*}} \sqrt{\frac{t_f}{\delta^*}} > 1 \) if and only if \( \frac{1 + \delta^*}{\delta^*} > 1 \). This condition is clearly true unless \(-\frac{1}{2} < \delta^* < 0\), which we know is not feasible.

\(^{15}\)Equivalently, we can also write this solution as

\[
\frac{U' \left( W - \rho \left( c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{t_f}{\delta^*}} \right) \right) - \delta \left( c_L + \sqrt{\frac{t_f}{\delta^*}} \right)}{U' \left( W - \rho \left( c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{t_f}{\delta^*}} \right) \right)} = \frac{c_L + \left( \frac{1 + \delta^*}{\sqrt{\delta^*}} \right) \sqrt{\frac{t_f}{\delta^*}}}{c_L + \left( \frac{1 + \delta^*}{\sqrt{\delta^*}} \right) \sqrt{\frac{t_f}{\delta^*}}} > 1
\]
Given that there are, in fact, no real positive values of δ∗ such that the left hand side of the equation equals 1, it must be that, on the right hand side of the equality, the marginal utility in the "loss state" is greater than the expected marginal utility. This is the definition of partial insurance.

5.1.2 Is no insurance the solution even without government services?

The question now becomes whether no insurance is preferred to partial insurance (that is, we want to find the conditions under which the solution is to have δ = 1). To see whether it is a possibility, let us find the conditions under which the solution is to have δ = 1. When evaluating the first order condition at δ = 1, then if the derivative is positive then it would mean that an agent would prefer to have a δ that is greater than 1 (that is, having less insurance). Using equation (1), we recall that an agent would like to increase \( f \) if the derivative is positive then it would mean that an agent would prefer to have a δ that is greater than 1. If the utility function is more generally CRRA so that \( U(\omega) = \frac{1}{1-\gamma} \omega^{1-\gamma} \), then \( U'(\omega) = \omega^{-\gamma} \) so that the condition for no insurance to be optimal becomes

\[
\frac{(1 - \rho) c_L + (1 - \rho) \sqrt{tf}}{(1 - \rho) c_L + \left(\frac{1}{2} - \rho\right) \sqrt{tf}} > \frac{W^\gamma}{(W - c_L - \sqrt{tf})^\gamma} > 1
\]
Isolating $W$, we find condition

$$W > \frac{(c_L + \sqrt{tJ})^{\frac{\gamma + 1}{\gamma}}}{(c_L + \sqrt{tJ})^{\frac{1}{\gamma}} - \left(c_L + \frac{1}{2} \left(1 - \frac{2\rho}{1 - \rho}\right) \sqrt{tJ}\right)^{\frac{1}{\gamma}}}$$

for no insurance to be optimal. In the special case of the utility function is $U(\omega) = \ln(\omega)$, so that $\gamma = 1$, then for no insurance to be optimal the individual’s wealth needs to be greater than

$$W > 2 \frac{1}{\sqrt{tJ}} (1 - \rho) \left(\sqrt{tJ} + c_L\right)^2$$

It seems clearer here that some insurance is more likely to be optimal when, in addition to the agent being more risk averse, the travel cost, $t$, the long-term care suppliers’ cost of entry in the market (or equivalently the lower is the number of long-term care service providers), $f$, and the cost of services, $c_L$, are greater, and when the probability of needing any type of long-term care services, $\rho$, and the individual’s wealth, $W$, are lower.

### 5.1.3 Insurance coverage with government services

Assuming that that $\delta^* \leq 1$ is the optimal level of insurance protection chosen by individuals, then the proportion of agents who will prefer government services, even in the presence of private long-term care insurance, is given by

$$z^* = 1 + \frac{2\pi}{f}R_L \left(\sqrt{\frac{ft}{\delta^*}} + \left(1 - \frac{\gamma}{\delta^*}\right)c_L - \frac{1}{\delta^*} \left(\frac{1}{\sin \frac{\pi}{2}}\right)R_L\right)$$

Clearly the demand for government services will depend on the optimal insurance choice\(^{16}\) and, perhaps more importantly, insurance choice will depend on the provision of government services. Agents should in fact anticipate that they are not certain to use the private long-term care services even if they have long-term care needs, and would therefore not collect any insurance benefit. This means that the agents’ maximization problem should include an endogenous probability that they will not use insurance. Given the impact of a coinsurance contract on the likelihood of asking for government services, we know that with probability $z^* (R_L, \delta^*)$ the agent will prefer entering a government facility over privately provided long-term care services. Using again $W$ as the individuals’ endowed wealth, the agent’s problem, assuming

\(^{16}\)Recall that

$$\frac{\partial z^*}{\partial \delta^*} = \frac{2\pi}{f (\delta^*)^2} R_L \left(-\frac{1}{2} \sqrt{\delta^* t} + \gamma c_L + \left(\frac{1}{\sin \frac{\pi}{2}}\right) R_L\right)$$

which would be positive, meaning that government services are more popular ($z^*$ increases) when insurance coverage decreases ($\delta$ increases), when

$$\delta < \frac{4}{ft} \left[\gamma c_L + \left(\frac{1}{\sin \frac{\pi}{2}}\right) R_L\right]^2$$

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von Neumann-Morgenstern expected utility over final wealth, is

\[
\max_{\delta^*} EU (\bullet) = (1 - \rho) U (W - \pi^{**}) + \rho \sum_{L=1}^{m} \xi_L U \left( W - \pi^{**} - \delta^* \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) \right) (1 - z^*(R_L, \delta^*)) \\
+ \rho \sum_{L=1}^{m} \xi_L U \left( W - \pi^{**} - \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right) \right) (z^*(R_L, \delta^*))
\]

subject to

\[
\pi^{**} = \rho \sum_{L=1}^{m} \xi_L (1 - \delta^*) \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) (1 - z^*(R_L, \delta^*))
\]

and

\[
z^*(R_L, \delta^*) = 1 + \frac{2\pi}{f} R_L \left( \sqrt{\frac{ft}{\delta^*}} + \left( 1 - \frac{\gamma}{\delta^*} \right) c_L - \frac{1}{\delta^*} \left( \frac{1}{\sin \frac{\delta}{2}} \right) R_L \right)
\]

Again, we see that full insurance (i.e., \(\delta^* = 0\)) is not optimal here since it would mean that the premium would be infinite.

The first order condition with respect to \(\delta^*\) is

\[
0 = -(1 - \rho) U' (W - \pi^{**}) \left( \frac{\partial \pi^{**}}{\partial \delta^*} \right) - \rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^{**} - \delta^* \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) \right) \left( \frac{\partial \pi^{**}}{\partial \delta^*} + \left( c_L + \frac{1}{2} \sqrt{\frac{tf}{\delta^*}} \right) \right) (1 - z^*(R_L, \delta^*)) \\
- \rho \sum_{L=1}^{m} \xi_L \left[ U \left( W - \pi^{**} - \delta^* \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) \right) - U \left( W - \pi^{**} - \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right) \right) \right] \frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*} \\
- \rho \sum_{L=1}^{m} \xi_L U' \left( W - \pi^{**} - \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right) \right) \left( \frac{\partial \pi^{**}}{\partial \delta^*} \right) z^*(R_L, \delta^*)
\]

where

\[
\frac{\partial \pi^{**}}{\partial \delta^*} = -\rho \sum_{L=1}^{m} \left[ \xi_L c_L + \left( \frac{1 + \delta^*}{2\delta^*} \right) \sqrt{\frac{tf}{\delta^*}} \right] (1 - z^*(R_L, \delta^*)) - \rho \sum_{L=1}^{m} \xi_L (1 - \delta^*) \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) \frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*}
\]

and

\[
\frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*} = \frac{2\pi}{f (\delta^*)^2} R_L \left( -\frac{1}{2} \sqrt{\delta^*} ft + \gamma c_L + \left( \frac{1}{\sin \frac{\delta}{2}} \right) R_L \right)
\]

If there is only one possible loss,

\[
0 = -(1 - \rho) U' (W - \pi^{**}) \left( \frac{\partial \pi^{**}}{\partial \delta^*} \right) \\
- \rho U' \left( W - \pi^{**} - \delta^* \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) \right) \left( \frac{\partial \pi^{**}}{\partial \delta^*} + \left( c_L + \frac{1}{2} \sqrt{\frac{tf}{\delta^*}} \right) \right) (1 - z^*(R_L, \delta^*)) \\
- \rho \left[ U \left( W - \pi^{**} - \delta^* \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) \right) - U \left( W - \pi^{**} - \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right) \right) \right] \frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*} \\
- \rho U' \left( W - \pi^{**} - \left( \frac{R_L}{\sin \frac{\delta}{2}} + \gamma c_L \right) \right) \left( \frac{\partial \pi^{**}}{\partial \delta^*} \right) z^*(R_L, \delta^*)
\]
where
\[
\frac{\partial \pi^{**}}{\partial \delta} = -\rho \left[ c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{tf}{\delta^*}} \right] (1 - z^*(R_L, \delta^*)) - \rho (1 - \delta^*) \left( c_L + \sqrt{\frac{tf}{\delta^*}} \right) \frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*},
\]
\[
z^*(R_L, \delta^*) = 1 + \frac{2\pi}{f} R_L \left( \sqrt{\frac{ft}{\delta^*}} + \left( 1 - \frac{\gamma}{\delta^*} \right) c_L - \frac{1}{\delta^*} \left( \frac{1}{\sin \frac{\pi}{2}} \right) R_L \right)
\]

and
\[
\frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*} = \frac{2\pi}{f (\delta^*)^2} R_L \left( -\frac{1}{2} \sqrt{\delta^* ft + \gamma c_L} + \left( \frac{1}{\sin \frac{\pi}{2}} \right) R_L \right)
\]

Suppose for simplicity that \( \frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*} = 0 \) (that is, having insurance have no impact, marginally, on the probability of getting services from the government). The problem then becomes

\[
0 = - (1 - \rho) U' (W - \pi^{**}) \left( \frac{\partial \pi^{**}}{\partial \delta^*} \right)
\]

\[
- \rho U' \left( W - \pi^{**} - \left( \frac{R_L}{\sin \frac{\pi}{2} + \gamma c_L} \right) \left( \frac{\partial \pi^{**}}{\partial \delta^*} \right) \right) z^*(R_L, \delta^*)
\]

\[
- \rho U' \left( W - \pi^{**} - \delta^* \left( c_L + \left( \frac{tf}{\delta^*} \right) \right) \left( \frac{\partial \pi^{**}}{\partial \delta^*} \right) \right) (1 - z^*(R_L, \delta^*))
\]

\[
- \rho U' \left( W - \pi^{**} - \delta^* \left( c_L + \left( \frac{tf}{\delta^*} \right) \right) \right) \left( c_L + \frac{1}{2} \sqrt{\frac{tf}{\delta^*}} \right) (1 - z^*(R_L, \delta^*))
\]

\[
- \rho \left[ U \left( W - \pi^{**} - \delta^* \left( c_L + \left( \frac{tf}{\delta^*} \right) \right) \right) - U \left( W - \pi^{**} - \left( \frac{R_L}{\sin \frac{\pi}{2} + \gamma c_L} \right) \right) \right] \frac{\partial z^*(R_L, \delta^*)}{\partial \delta^*}
\]

where

\[
\frac{\partial \pi^{**}}{\partial \delta^*} = -\rho \left[ c_L + \left( \frac{1 + \delta^*}{2 \delta^*} \right) \sqrt{\frac{tf}{\delta^*}} \right] (1 - z^*(R_L, \delta^*))
\]

\[
z^*(R_L, \delta^*) = 1 + \frac{2\pi}{f} R_L \left( \sqrt{\frac{ft}{\delta^*}} + \left( 1 - \frac{\gamma}{\delta^*} \right) c_L - \frac{1}{\delta^*} \left( \frac{1}{\sin \frac{\pi}{2}} \right) R_L \right)
\]

6 Family help

We now introduce the contribution of one’s family to the provision of long-term care services. We shall define families, in our model, according to two family-specific parameters: the distance (emotionnally or physically) they are from the elderly individual, which we denote \( H_i \), and their cost premium of providing help compared to the private sector, which we denote \( \eta_i \). As we see, the provision of family help is modelled similar to the provision of government services, with the difference that family help is agent-specific. Let \( h_i \in (0, 180) \) – in degrees – represent this help\(^{17}\) for individual \( i \) such that the distance that this individual has to travel to

\(^{17}\text{We can think that } h_i \text{ increases with the number of children, with the number of female children, with the number of children living close by, with the number of children having formal training in health care, with the number of children unemployed (so that the opportunity cost of help is small), etc...}
seek family help given health condition $R_L$ is $H^i_L = R_L \left( \sin \left( \frac{h^i}{2} \right) \right)^{-1}$. We first examine the impact of family when there is no long-term care insurance market (so that $\delta = 1$), and then we will introduce insurance. Of particular interest in this section is how the market is segmented according to individual preferences for informal family care, government provided long-term care, and privately run long-term care facilities.

6.1 Without insurance

6.1.1 Family versus private

Using the same approach as before, but letting $h^i$ represent an individual’s "fall back" to family help rather than the private provision of LTC services, we find that an agent in the economy, for a given illness level $R_L$, will choose to let his family take care of him provided that $p^*_L = c_L + \sqrt{f_j} \geq \eta^i c_L + \left( \frac{1}{\sin \frac{h^i}{2}} - 2\pi \left( \frac{1}{\eta^i} \right)^2 t \right) R_L$, where $\eta^i > 0$ is the cost premium to the family (similar to the cost premium of government services, $\gamma$). For any agent’s family, its cost of providing informal help can be seen as being embedded in the two variables $\eta^i$ and $h^i$ so that the greater the family’s opportunity cost, the greater is $\eta^i$, and the "further away" is the family, the smaller is $h^i$. Substituting for $n^*_L = 2\pi R_L \sqrt{f}$ gives us that an agent will prefer his family’s informal help to the formal help of a private provider of long-term care services if and only if

$$
(1 - \eta^i) c_L + \sqrt{f_j} \geq \frac{1}{\sin \frac{h^i}{2}} R_L - \frac{f}{2\pi R_L}
$$

Informal family help will therefore be chosen provided that

$$\sin \frac{h^i}{2} \geq \frac{2\pi (R_L)^2}{((1 - \eta^i) c_L + \sqrt{f_j}) 2\pi R_L + f}$$

We thus have that family help is more likely to be prefered by some agent when his illness is not severe (i.e., $R_L$ is small), when the market for the private provision of LTC services is not very competitive (i.e., $f$ is large), when the travel cost to the private facilities is high (i.e., $t$ is large), the closer is one’s family (i.e., $h^i$ is large), and the less costly for the family is their time invested in the elderly parent (i.e., $\eta^i$ is small). Using the same approach as in the comparison between the government and the private sector, we can show that the proportion of individuals who would prefer to receive informal family help to formal privately provided long-term care services is given by

$$z^I = 1 + \frac{2\pi}{f} R_L \left( \sqrt{f_j} + (1 - \eta^i) c_L - \frac{1}{\sin \frac{h^i}{2}} R_L \right),$$

where the superscript $I$ refers to the informality of family help. This proportion does not only depend on the elderly’s individual’s "draw" of his needs with activities of daily living, it also depends on his preference for and his family’s ability to provide informal care.
6.1.2 Family versus public

The choice between informal family help and the public provision of LTC services will be given by the following simple decision rule: Informal help is marginally preferred if and only if

\[ i = \frac{1}{\sin \frac{R}{c_L} + g} \leq \gamma c_L + \frac{1}{\sin \frac{R}{c_L} + \frac{g}{2}} R_L. \]

It may be easier to examine the relationship if we were to rewrite it as

\[ (\eta^i - \gamma) \leq \left( \frac{\sin \frac{h^i}{2} - \sin \frac{g}{2}}{\sin \frac{h^i}{2} + \sin \frac{g}{2}} \right) R_L. \]

If \( \eta^i \leq \gamma \) and \( h^i \geq g \), meaning that the cost to the family of informal help is lower than the cost of obtaining services from the government, and the family is "closer" (either physically or emotionally or psychologically) than the government, then for sure an individual will prefer informal family help. Inversely, if \( \eta^i \geq \gamma \) and \( h^i \leq g \), then government services would be preferred.

6.1.3 Segmenting the market between family help, public services, and private provision

At this point of the paper, we realize that elderly individuals have the choice of obtaining services from the three aforementioned providers: family, government, and private. We now want to explain the attractiveness of each of these providers and how much "market share" of long-term care services they have on the market. We recall that, without any insurance, the proportion of individuals who prefered obtaining services from the government than the private sector was given by

\[ z^0 = 1 + \frac{2\pi}{f} R_L \left( \sqrt{f^2 - (1 - \gamma) c_L - \frac{1}{\sin \frac{\gamma}{2} R_L}} \right). \]

We know that the proportion of individuals who prefer to receive informal family help to formal privately provided long-term care services is given by \( z^I \), which may be larger or smaller than \( z^0 \) depending on the individual. Nonetheless, we know that the maximum proportion of long-term care services that will be provided by the private sector is \( \min \{1 - z^I, 1 - z^0\} \). Suppose that \( z^I > z^0 \), which would mean that a larger proportion of individuals prefer informal help to privately provided long-term care services than prefer government services to private services.

6.2 With coinsurance

When we introduce insurance, the choice of informal family help over the private provision of long-term care services is still dictated by the relative cost of one compared to the other. The proportion of individuals who would prefer informal family care over partially-insured private long-term care services would be given by

\[ z^{IF} = 1 + \frac{2\pi}{f} R_L \left( \sqrt{\frac{f}{\delta^2} + \left(1 - \frac{\eta^i}{\delta^2}\right) c_L - \frac{1}{\sin \frac{1}{2} \delta^2} \left( \frac{1}{\sin \frac{\gamma}{2} \delta^2} \right) R_L} \right). \]
7 Political economy (or choosing g)

We did not discuss how $g$ was chosen. We will suppose that agents "vote" on $g$. Starting with the case of no informal family help, we will find the $g$ which, financed with break even taxes, maximizes the agents' expected utility independent of the insurance coverage first.

8 Conclusion (to be written)

9 References (to be completed)